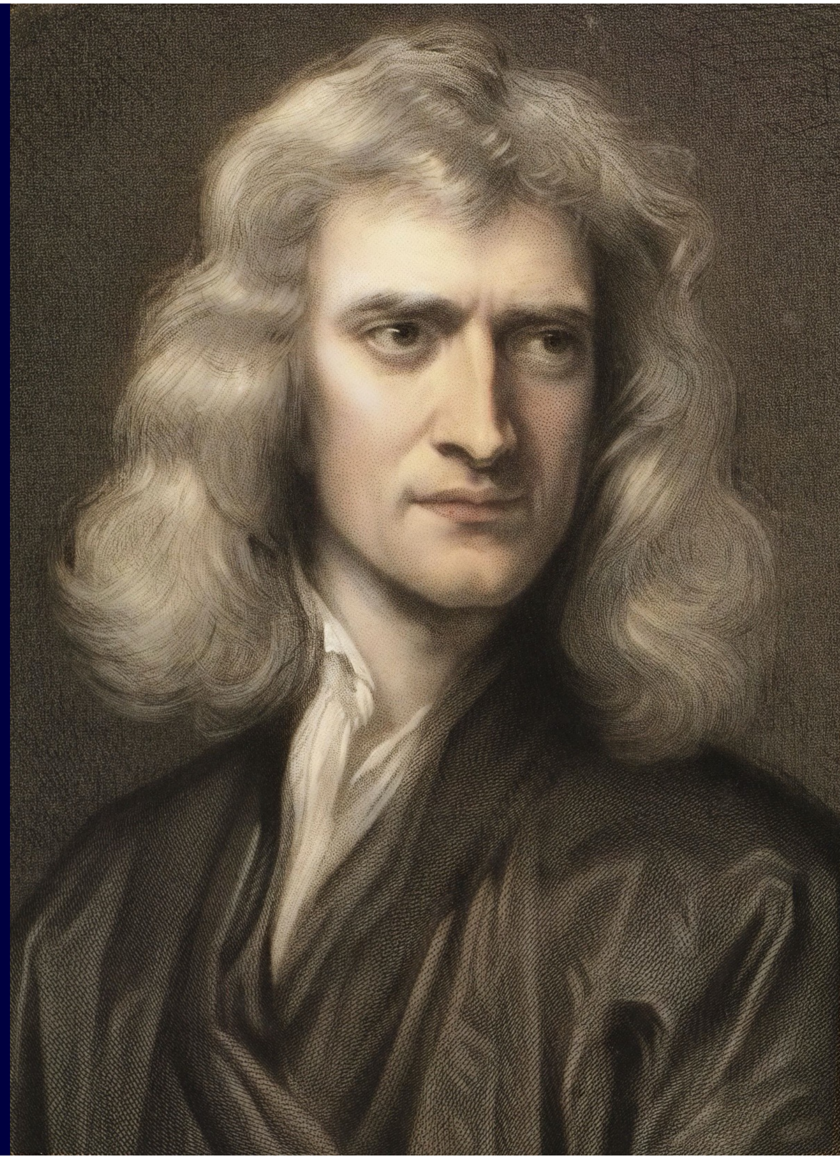
Physics 2A: Lecture 10

Today's Agenda

- Impulse and Momentum

(or the chapter where physicists run out of letters)

  - Non-constant forces
  - Impulse-momentum thm
- Conservation of Linear momentum
  - External/Internal forces
  - Examples



Linear Momentum

- Moving object has linear momentum such that:

$$\vec{p} = m\vec{v}$$

- Vector quantity in same direction as velocity

$$p_x = mv_x$$

- Units: kg m/s or N s

$$p_y = mv_y$$

What makes change in motion difficult?
Momentum gives a sense of how hard it is to stop an object
Student, "I really don't understand what exactly momentum is interms of a quantity."

Newton's Second law

- Newton's second law was originally written in terms of momentum

m is constant

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} \quad (1)$$

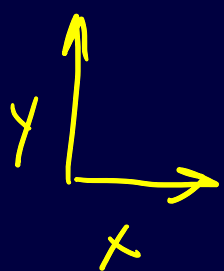
$$\vec{F}_{net} = \frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

$$\vec{F}_{net} = m\vec{a} \quad (2)$$

- Equation (1) and (2) are equivalent if the mass is not allowed to change.

Newton's Second Law

- We can rearrange Newton's second law



$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

$$J_x = \Delta p_x$$

$$J_y = \Delta p_y$$

$$d\vec{p} = \vec{F}_{net}\ dt$$

$$\int_{\vec{p}_i}^{\vec{p}_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}_{net}\ dt$$

$$\vec{p}_f - \vec{p}_i = \int_{t_i}^{t_f} \vec{F}_{net}\ dt = \vec{J}$$

→ Impulse

Impulse-Momentum Theorem

- An impulse on an object leads to a change in the objects momentum. (This is a vector)

$$J_x = \Delta p_x$$

$$J_x = p_{fx} - p_{ix}$$

$$p_{fx} = p_{ix} + J_x$$

Student, “Why in the textbook the formula J = pf - pi is written in scalars (not in vectors)?”

- The momentum ‘after’ an interaction, like a collision or explosion, equals the momentum before the interaction plus the impulse that arises from the interaction.

Momentum can be positive or negative!!

Clicker Question 1:

You drop an egg onto A) the floor B) a thick piece of foam rubber. In both cases, the egg does not bounce (assume the egg hits each surface with the same speed v).

In which case is the magnitude of the impulse caused by the collision with the surface greater?

A) Floor
B) Foam
C) It's the same

Clicker Question 1:

You drop an egg onto A) the floor B) a thick piece of foam rubber. In both cases, the egg does not bounce (assume the egg hits each surface with the same speed v).

$$J_y = p_{fy} - p_{iy}$$

$$J_y = 0 - m(-v)$$

$$J_y = mv$$

$$F_{avg\,y}\Delta t = mv$$

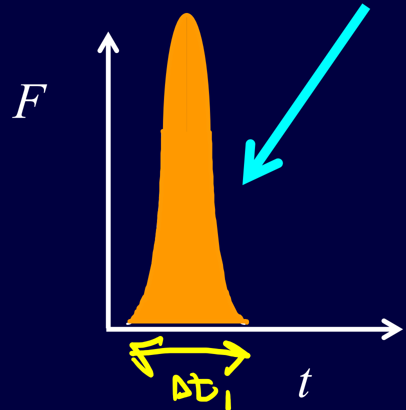
$$F_{avg\,y} = \frac{mv}{\Delta t}$$

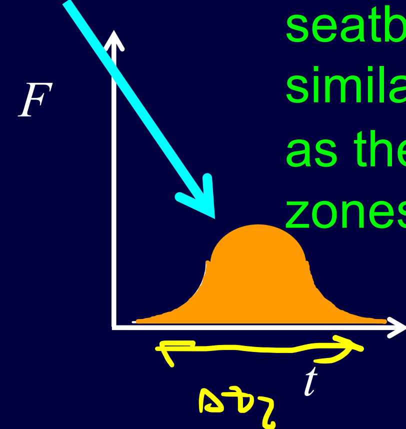
Smaller Δt = larger F

Pre-Class Quiz

- This is why cars have crumple zones, to increase Δt and decrease the force.

Same Area = Impulse





Student: Do airbags and seatbelts do similar things as the crumple zones on cars?

$$\textbf{impulse} = J_x \equiv \int_{t_i}^{t_f} F_x(t)\,dt$$

$$= \text{area under the } F_x(t) \text{ curve between } t_i \text{ and } t_f$$

Clicker Question 2:

Two identical balls are dropped from the same height onto the floor. In case 1 the ball bounces back up, and in case 2 the ball sticks to the floor without bouncing. In which case is the impulse given to the ball by the floor the biggest?

(a) Case 1

(b) Case 2

(c) Both are the same

Clicker Question 2:

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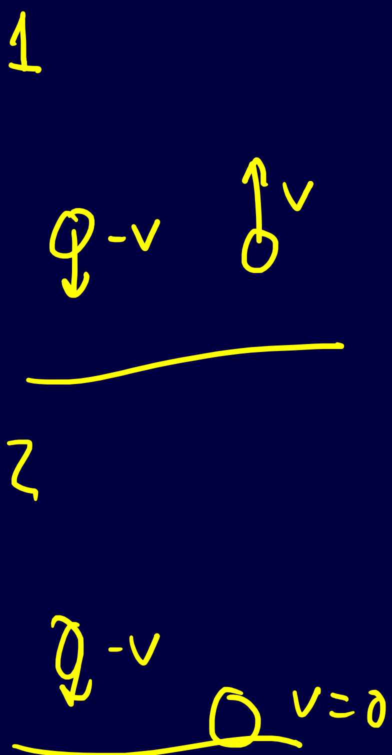


Case 1: $J_y = p_{fy} - p_{iy}$

$$J_y = mv - m(-v)$$

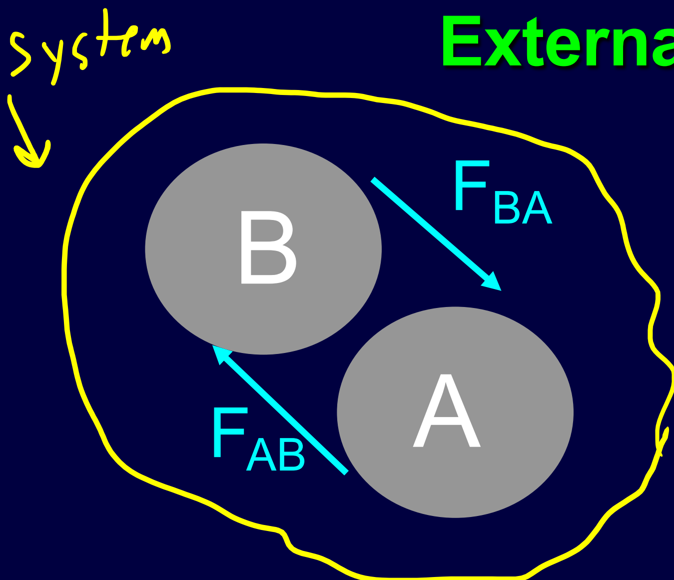
$$J_y = 2mv$$

Case 2: $J_y = p_{fy} - p_{iy}$

$$J_y = 0 - m(-v)$$

$$J_y = mv$$

External/Internal Forces



N3 Law

$|F_{AB}| = |F_{BA}|$

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

$$\vec{F}_{net}^{ext} + \vec{F}_{net}^{\text{int}} = \frac{d\vec{p}}{dt}$$

$$\vec{F}_{net}^{ext} + 0 = \frac{d\vec{p}}{dt}$$

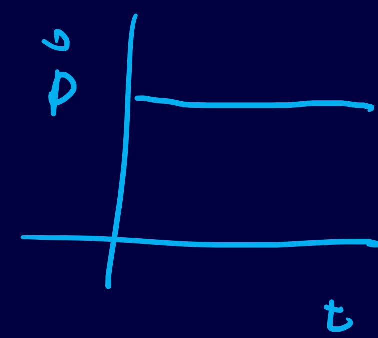
for 1 system $\rightarrow$ $\vec{F}_{net}^{ext} = \dfrac{d\vec{p}}{dt}$

Conservation of Momentum

- Newton's second law for a system

$$\vec{F}_{net}^{ext} = \frac{d\vec{p}}{dt}$$

$$0 = \frac{d\vec{p}}{dt}$$



- The total momentum of an isolated system remains constant
  - Isolated system = no external forces

Conservation of Linear Momentum

- The total momentum of an isolated system remains constant

$$\vec{p}_f = \vec{p}_i$$

$$\Sigma m\vec{v}_f = \Sigma m\vec{v}_i$$

$$\Sigma mv_f = \Sigma mv_i$$ in x-direction

$$\Sigma mv_f = \Sigma mv_i$$ In y-direction

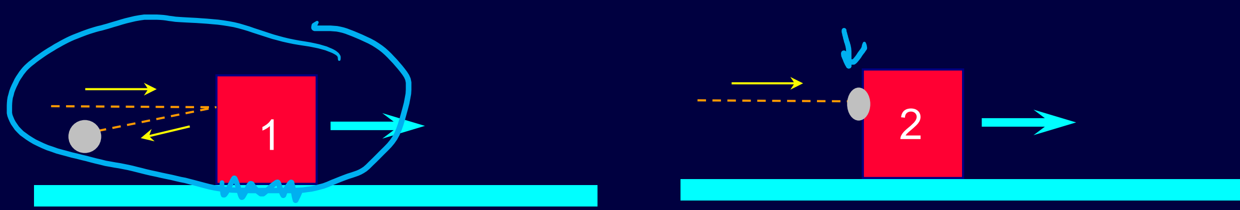
only 2 objects

object 1		object 2		object 1		object 2
mv_f	+	mv_f	=	mv_i	+	mv_i

Clicker Question 3:

- Two balls of equal mass are thrown horizontally with the same initial velocity. They hit identical stationary boxes resting on a frictionless horizontal surface.
- The ball hitting box 1 bounces back, while the ball hitting box 2 gets stuck.
  - Which box ends up moving faster?

(a) **Box 1** (b) **Box 2** (c) **same**



Impulse delivered to box 1 is larger

1

$p_i^x = p_f^x$

$5 = -5 + p_B$

$p_B = 10$

2

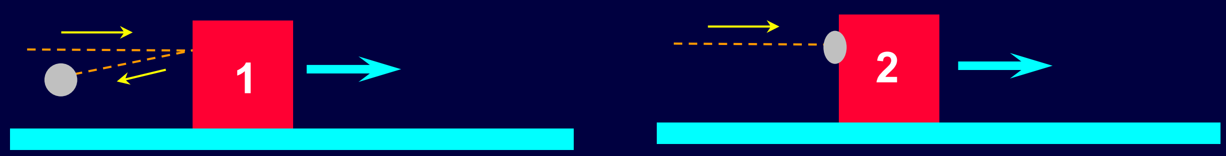
$p_i^x = p_f^x$

$5 = 1 + p_B$

$p_B = 4$

Clicker Question 3:

- Since the total external force in the x-direction is zero, momentum is conserved along the x-axis.
- In both cases the initial momentum is the same (*mv* of ball).
- In case 1 the ball has negative momentum after the collision, hence the box must have more positive momentum if the total is to be conserved.
- The speed of the box in case 1 is greater.



Noether's Thm

- Emmy Noether developed the general connection between symmetry and conservation laws
- Symmetries describe changes that can be made without altering how an object looks or acts. A sphere is perfectly symmetric: Rotate it any direction and it appears the same. Likewise, symmetries pervade the laws of physics: Equations don't change in different places in time or space.
- Noether's theorem proclaims that every such symmetry has an associated conservation law, and vice versa — for every conservation law, there's an associated symmetry.

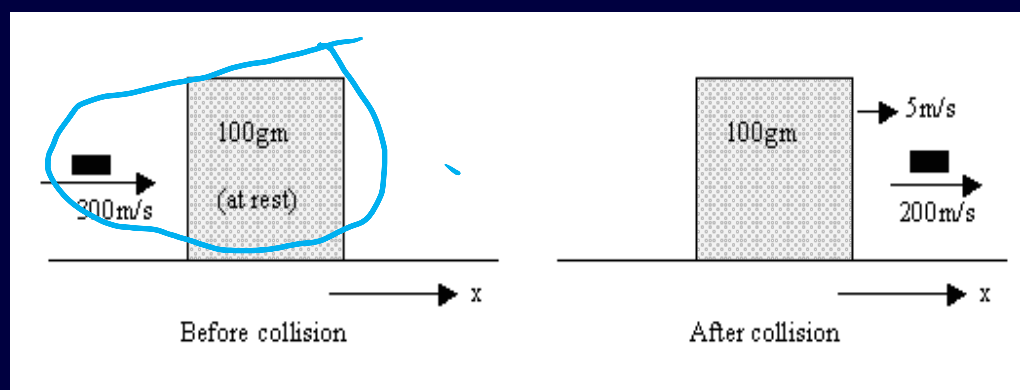
- ***Emmy Noether's Wonderful Theorem,***

Dwight E. Neuenschwander

Clicker Question 4:

A bullet having an initial velocity of 300 m/s in the +x direction penetrates an initially stationary soda can of mass 100 gm and emerges on the other side with a final velocity of 200 m/s in the +x direction. The velocity of the soda can after the collision is 5 m/s, also in the +x direction. Assume the soda can slides on a horizontal frictionless surface. What is the mass of the bullet?

A. 2 gm
B. 5 gm
C. 12 gm
D. 21 gm
E. 25 gm

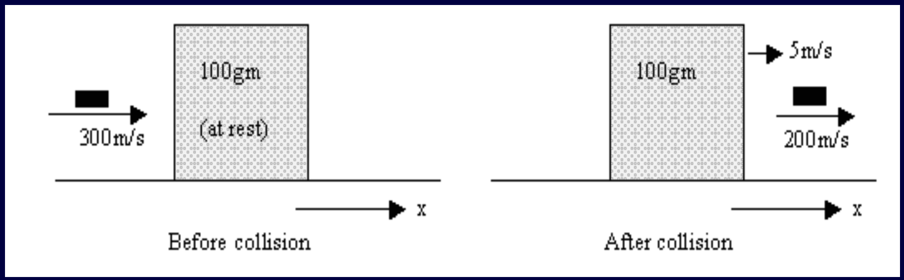


* system: CAN Bullet
no external forces in x-direction

Clicker Question 4:

$m_1 \rightarrow$ bullet

$m_2 \rightarrow$ can



$$P_i^x = P_f^x$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$m_1 (300 \text{ m/s}) = m_1 (200 \text{ m/s}) + (100 \text{ gm}) 5 \text{ m/s}$$

$$m_1 \, 300 \text{ m/s} = m_1 \, 200 \text{ m/s} + 500 \text{ gm m/s}$$

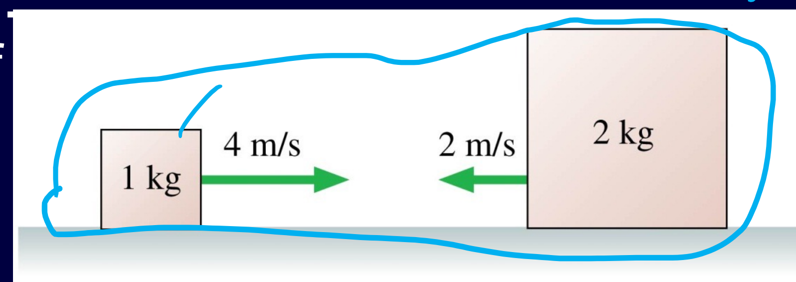
$$300 m_1 = 200 m_1 + 500 \text{ gm}$$

$$100 m_1 = 500 \text{ gm}$$

$$\boxed{m_1 = 5 \text{ gm}}$$

Clicker Question 6:

The two boxes are sliding along a frictionless surface. They collide and stick together. Afterward, the velocity of the two boxes is



A. 2 m/s to the left.

B. 1 m/s to the left.

C. 0 m/s, at rest.

D. 1 m/s to the right.

E. 2 m/s to the right.

No external forces in x-dir
Momentum is conserved in x-direction

+x →

$$p_i^x = p_f^x$$

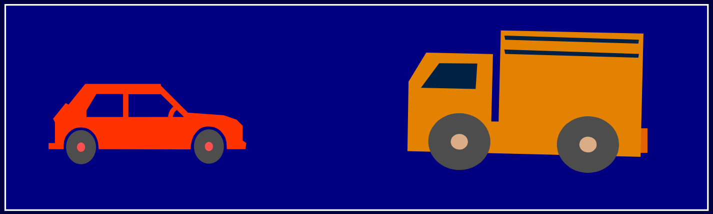
$$(1\,\text{kg})\,4\,\text{m/s} + 2\,\text{kg}(-2\,\text{m/s}) = 3\,\text{kg}(v_f)$$

$$4\,\text{kg m/s} - 4\,\text{kg m/s} = 0 = 3\,\text{kg}\,v_f$$

$$v_f = 0$$

Clicker Question 5:

A small car and a large truck collide head-on and stick together. Which one feels the larger impulse?



a) the car

b) the truck

c) they both feel the same impulse

d) can't tell without knowing the final velocities

Clicker Question 5:

A small car and a large truck collide head-on and stick together. Which one feels the larger impulse?

$$J_x = F_x \Delta t$$

Follow-up: Which one feels the larger acceleration?

Collisions and Explosions

- Conservation of momentum is very useful for analyzing collisions and explosions
- In collisions/explosions, forces are very complicated; momentum gives us a useful way to solve these problems (treat colliding/exploding particles as system)
- Momentum conservation is almost always a very good model to use during a collision/explosion (as the external forces are generally small compared to collision/explosion forces)
- This is one reason why momentum is so important

$$F_{avg\,y}\Delta t = \Delta p$$

Conservation of Linear Momentum

The total momentum of an isolated system remains constant

$$\vec{P}_i = \vec{P}_f$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

If the motion is along a single axis, the motion is 1-d and we can write the equation about that axis:

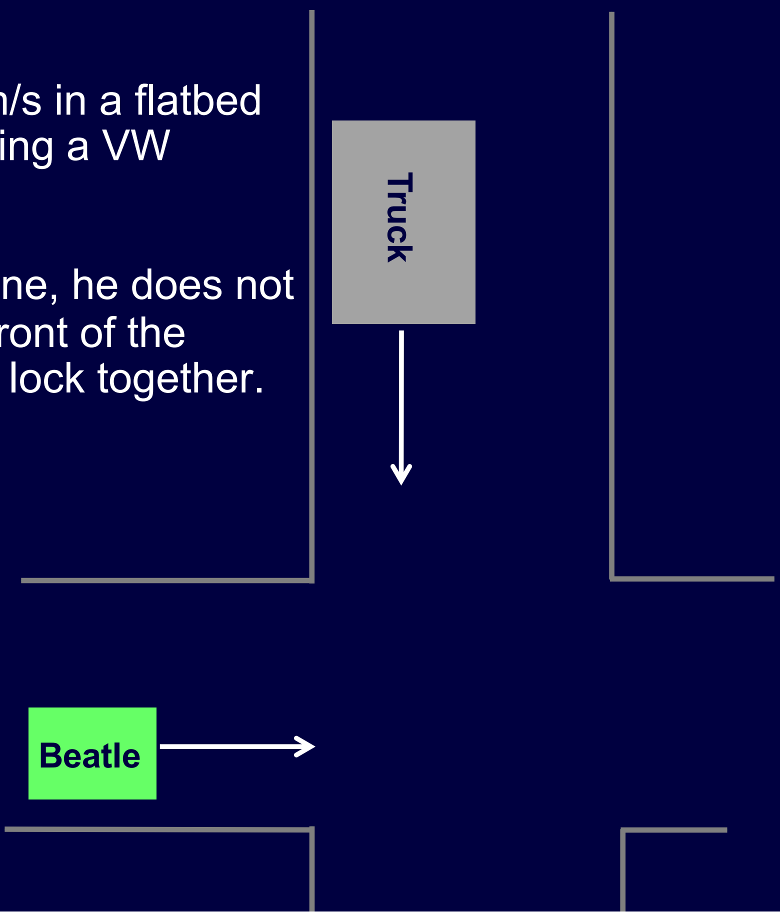
$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

Example: a 2-D Collision

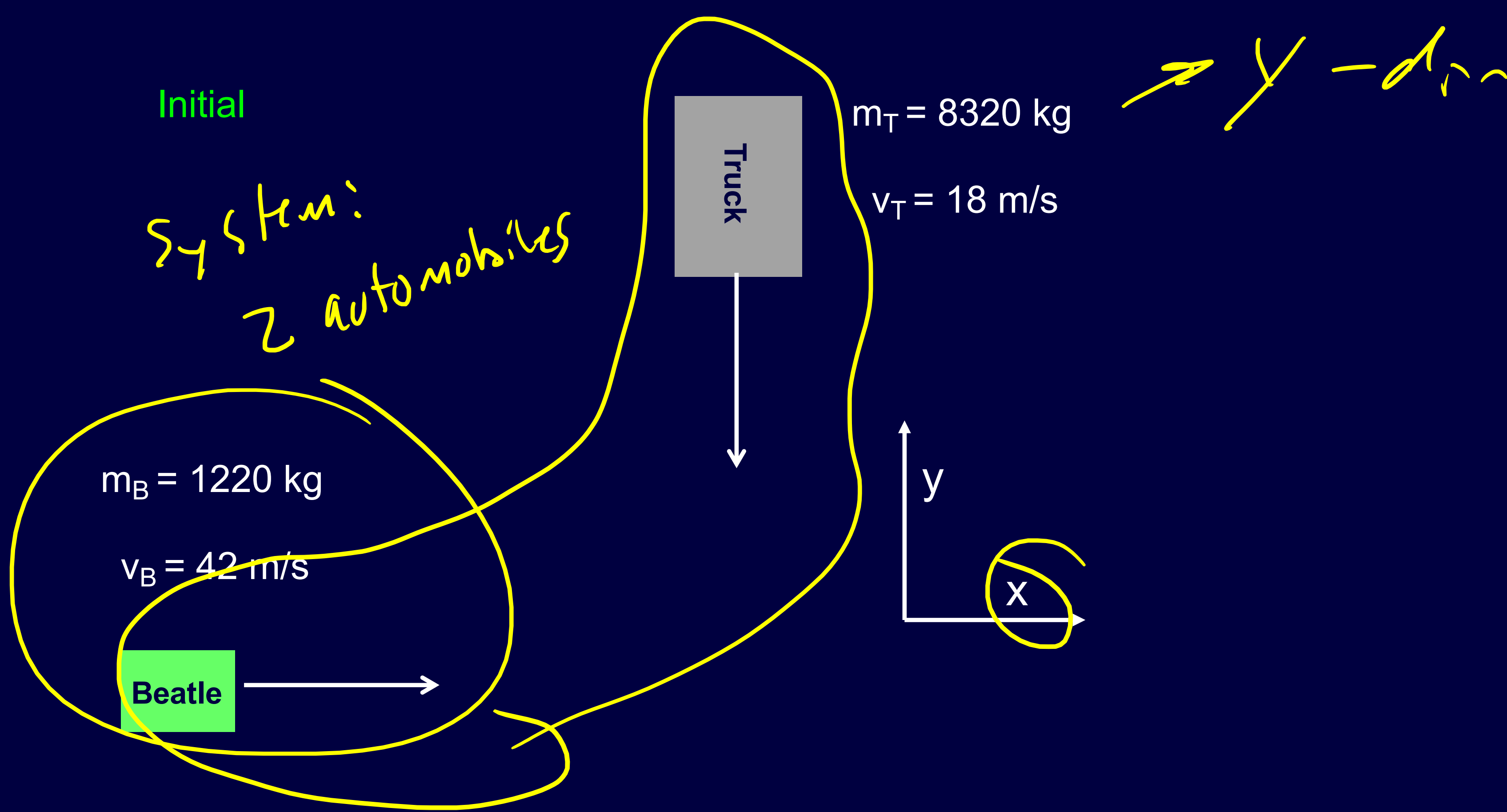
George is driving south along at 18.0 m/s in a flatbed truck heavily loaded, while John is driving a VW Beatle at 42.0 m/s east.

Because John is texting on his cell phone, he does not see the red light and collides with the front of the truck. During the collision, the vehicles lock together.

If George's truck has a total mass of 8320 kg and John's Beatle has a total mass of 1220 kg, find the velocity of the interlocked vehicles immediately after the collision.



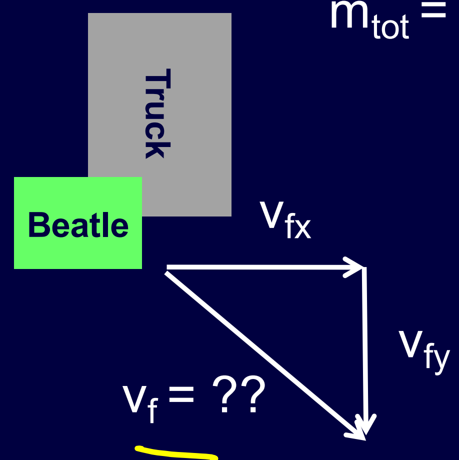
Example



Initial
System:
2 automobiles
Truck
m_T = 8320 kg
v_T = 18 m/s
→ y-dir
m_B = 1220 kg
v_B = 42 m/s
Beatle
y
x

Example

Final



m_{tot} = 9540 kg

Clicker Question 6:

After the accident, which is true about the angle of the velocity of the two cars relative to the eastward direction?

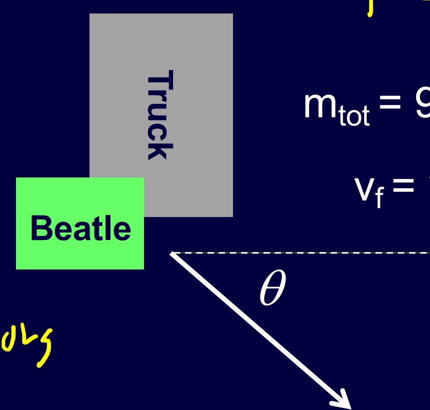
a) $\theta < 45°$

b) $\theta > 45°$

c) $\theta = 45°$

$m_T = 8370$ kg

$v_T =$ 18 m/s

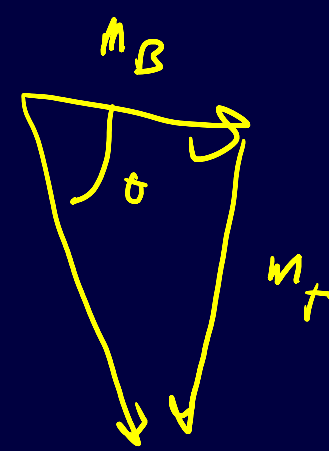


$m_{tot} = 9540$ kg

$v_f = ??$

$m_B = 1270$ kg

$v_B = 42$ m/s



Clicker Question 7:

What is the correct equation for the x-direction momentum?

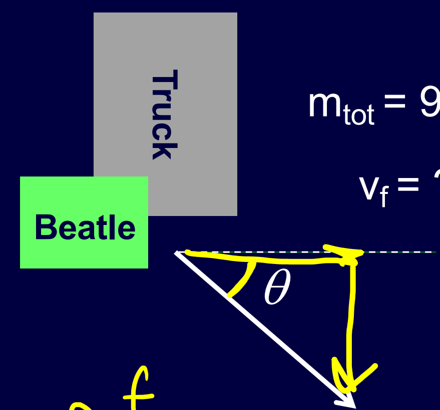
a) $m_B v_B = (m_T + m_B) v_{xf}$

b) $m_T v_T = (m_T + m_B)\, v_{xf}$

c) $m_B v_B + m_T v_T = (m_T + m_B)\, v_{xf}$

d) $m_B v_B = (m_T + m_B) v_f \, Sin\,\theta$

Cos θ



m_{tot} = 9540 kg

v_f = ??

$P_x^i = P_x^f$

Collision 3

x-direction

object 1 object 2 object 1 object 2

$$mv_i + mv_i = mv_f + mv_f$$

$$m_B v_B = m_B v_{xf} + m_T v_{xf}$$

$$m_B v_B = (m_B + m_T)\, v_{xf}$$

$$v_{xf} = \frac{m_B v_B}{(m_B + m_T)} = \frac{(1220\text{kg})(42\text{m/s})}{9540\text{ kg}} = 5.37\text{m/s}$$

Collision 3

y-direction

↓ +y

object 1	object 2	object 1	object 2

$$mv_i + mv_i = mv_f + mv_f$$

$$m_T v_T = m_B v_{yf} + m_T v_{yf}$$

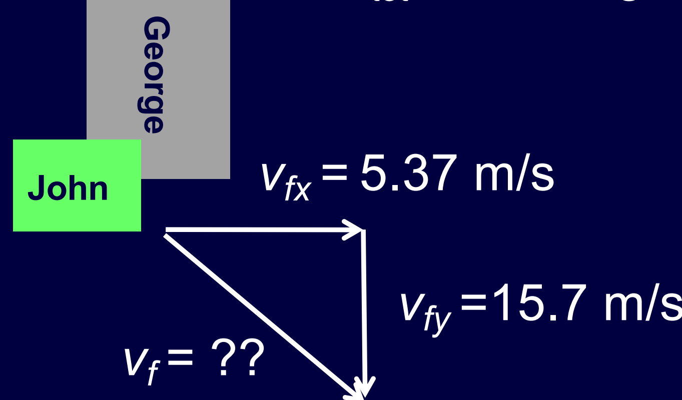
$$m_T v_T = (m_B + m_T)\, v_{yf}$$

$$v_{yf} = \frac{m_T v_T}{(m_B + m_T)} = \frac{(8320\,\text{kg})(18\,\text{m/s})}{9540\,\text{kg}} = 15.7\,\text{m/s}$$

Example

Final

m_{tot} = 9540 kg



$$v_f = \sqrt{(5.37m/s)^2 + (15.7m/s)^2} = 16.6m/s$$

$$\theta = \arctan\left(\frac{15.7}{5.37}\right) = 71^\circ$$

Clicker Question 8:

Two dry ice pucks slide on a horizontal, frictionless surface. Puck A, which is 2 kg, moves at speed 7 m/s in the positive *x* direction and Puck B, which is 1 kg, moves at speed 8 m/s in the negative y direction as shown in the figure. The pucks collide near the star in the figure. After the collision Puck B is moving in the positive x direction, while Puck A is moving in the direction that makes 45° with the positive *x*-axis as illustrated.

What is the speed of Puck A after the collision?

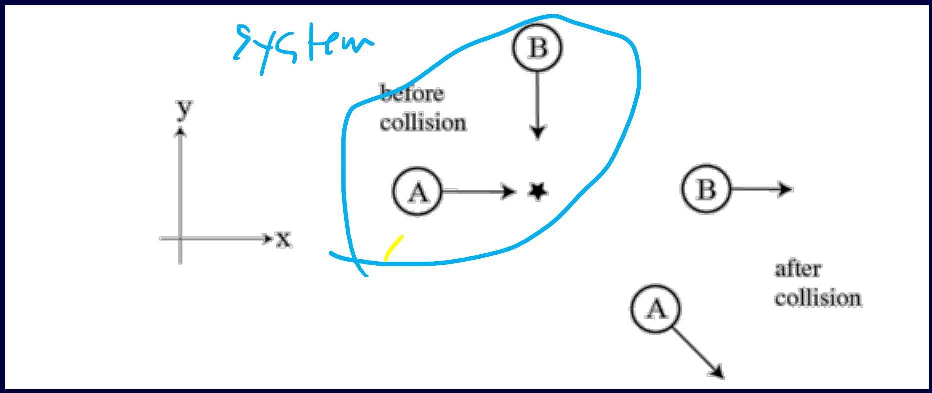
(a) 1 m/s

(b) 2 m/s

(c) 2√2 m/s

(d) 4 m/s

(e) 4√2 m/s



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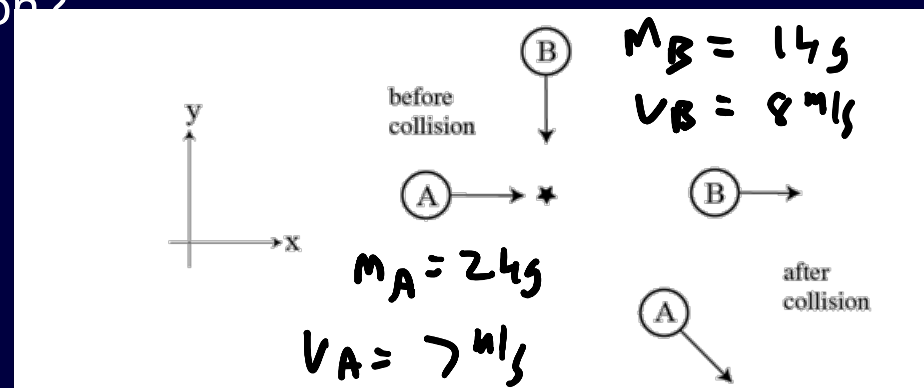

y-direction

$$P_i^y = P_f^y$$

$$m_B v_B = m_A v_{Af} \sin 45$$

$$v_{Af} = \frac{m_B v_B}{m_A \sin 45} = \frac{(1 kg)\, 8 \frac{m}{s}}{2 kg \sin 45}$$

$$= \frac{1\,(8\, m/s)}{2\, \frac{\sqrt{2}}{2}} = \frac{8}{\sqrt{2}}\, m/s \left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{8\sqrt{2}}{2}\, m/s = 4\sqrt{2}\, \frac{m}{s}$$

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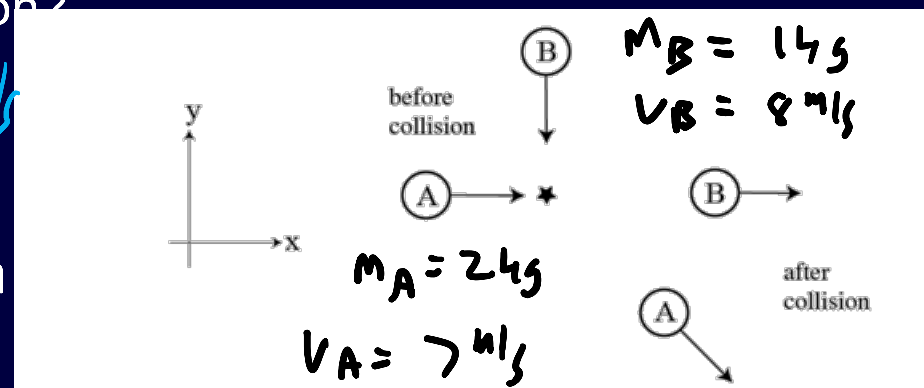

$m_B = 1kg$, $v_B = 8\,m/s$

$m_A = 2kg$, $v_A = 7\,m/s$

v_{Bf} | (A) $5\frac{m}{s}$ (B) $6\,m/s$ (C) $7\,m/s$

$$p_i^x = p_f^x$$

$$m_A v_A = m_B v_{Bf} + m_A v_{Af} \cos 45$$

$$v_{Bf} = \frac{1}{m_B}\left[m_A v_A - m_A v_{Af} \cos 45\right]$$

$$= \frac{1}{1kg}\left[2kg\,(7\,m/s) - 2kg\,4\sqrt{2}\,\frac{\sqrt{2}}{2}\right] = 6\,m/s$$