Today's Agenda

(1) Linear Momentum
(2) The Conservation of Linear Momentum
(3) 1-d Collisions

$F = ma$

Clicker Question 1:

Two blocks, one of mass M and the other of mass 2M, are on a horizontal frictionless surface and are initially at rest. Each block is now acted on by the same constant force F for the same time interval Δt. After the force acts, which block has the larger momentum?

(a) The block of mass M.

(b) The block of mass 2M.

(c) The two blocks have the same momentum.

$\vec{F} = m\vec{a}$

$\vec{a} = \frac{\vec{F}}{m}$

$\vec{J}_x = \boxed{F \Delta t} = \Delta p$

$= p_f - p_i$

Impulse

Clicker Question 2:

$m_{Bullet} = 0.005$ kg

Now compare the magnitude of the net impulse on the bullet, J_B, with the magnitude of the net impulse on the soda can, J_S, during the collision.

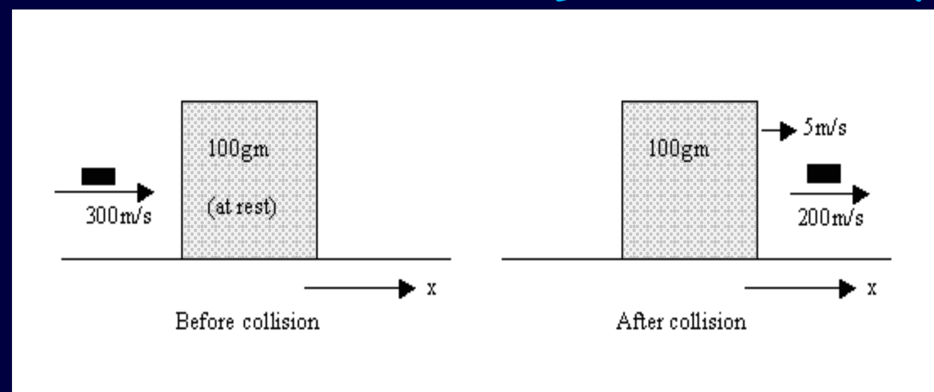
$m_{can} = 0.100$ kg

$|J_x| = \int F^x dt = F^x_{Avg} \Delta t$

A. $J_B > J_S$

B. $J_B = J_S$

C. $J_B < J_S$



$\Delta P_{can} = P_f - P_i = (0.100\text{ kg})[V_f - V_i] = 0.100\text{kg}[5\text{m/s} - 0] = \boxed{0.5\text{ kg m/s}}$

$\Delta P_{bullet} = P_f - P_i = [0.005\text{ kg}][V_f - V_i] = 0.005\text{kg}[200\text{m/s} - 300\text{m/s}] = \boxed{-0.5\text{ kg m/s}}$

Conservation of Linear Momentum

The total momentum of an isolated system remains constant

$$\vec{\mathrm{P}}_i = \vec{\mathrm{P}}_f$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

If the motion is along a single axis, the motion is 1-d and we can write the equation about that axis:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

Collisions and Explosions

- Conservation of momentum is very useful for analyzing collisions and explosions
- In collisions/explosions, forces are very complicated; momentum gives us a useful way to solve these problems (treat colliding/exploding particles as system)

$F \neq ma$

- Momentum is almost always conserved during a collision/explosion (external forces are generally small compared to collision/explosion forces)

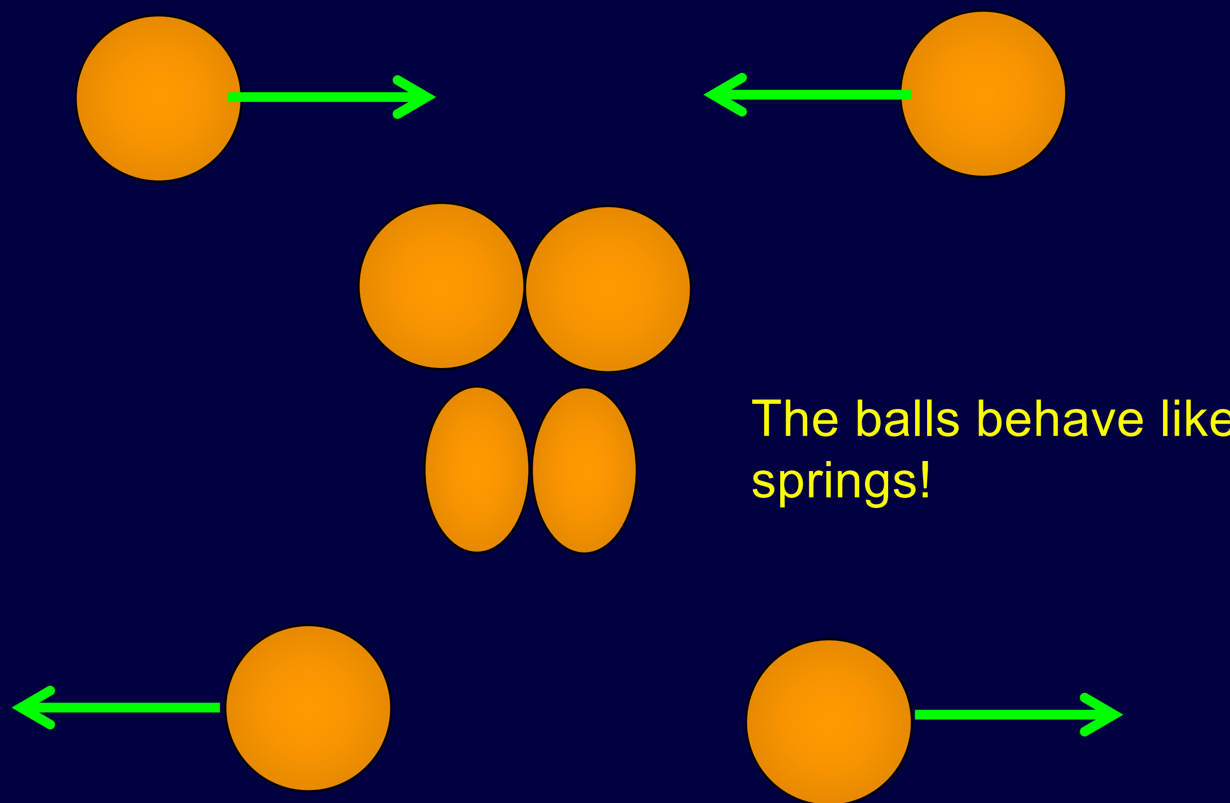
- This is one reason why momentum is so important ✓

$$F_{avg\,y}\Delta t = \Delta p \Rightarrow \text{Impulse}$$

Collisions

- Momentum is almost always conserved during as collision (external forces are generally small compared to collision forces)
- Two kinds of collisions
  - Elastic-KE is conserved → idealization special case
    - Very special case
  - Inelastic-KE is not conserved
    - Can be assumed if
      - Objects stick together ✓
      - Damage is done during collision ✓

Elastic Collision



Clicker Question 3:

Two perfect billiard balls collide on a flat, frictionless pool table.

Which below is true?

(A) Mechanical energy is conserved for the balls

(B) Linear momentum is conserved for the balls in the plane of the table

(C) Both (A) and (B)

(D) Neither (A) or (B)

(E) Neither (A) or (B) or (C)

Elastic Collision

Elastic

- During an elastic collision both momentum and mechanical energy are conserved:

mom

$$m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + m_2 v_{2i} \qquad (1)$$

KE

$$\tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2 = \tfrac{1}{2} m_1 v_{1i}^2 + \tfrac{1}{2} m_2 v_{2i}^2 \qquad (2)$$

- Combining equation (1) and (2) we get that "speed of approach equals speed of recession":

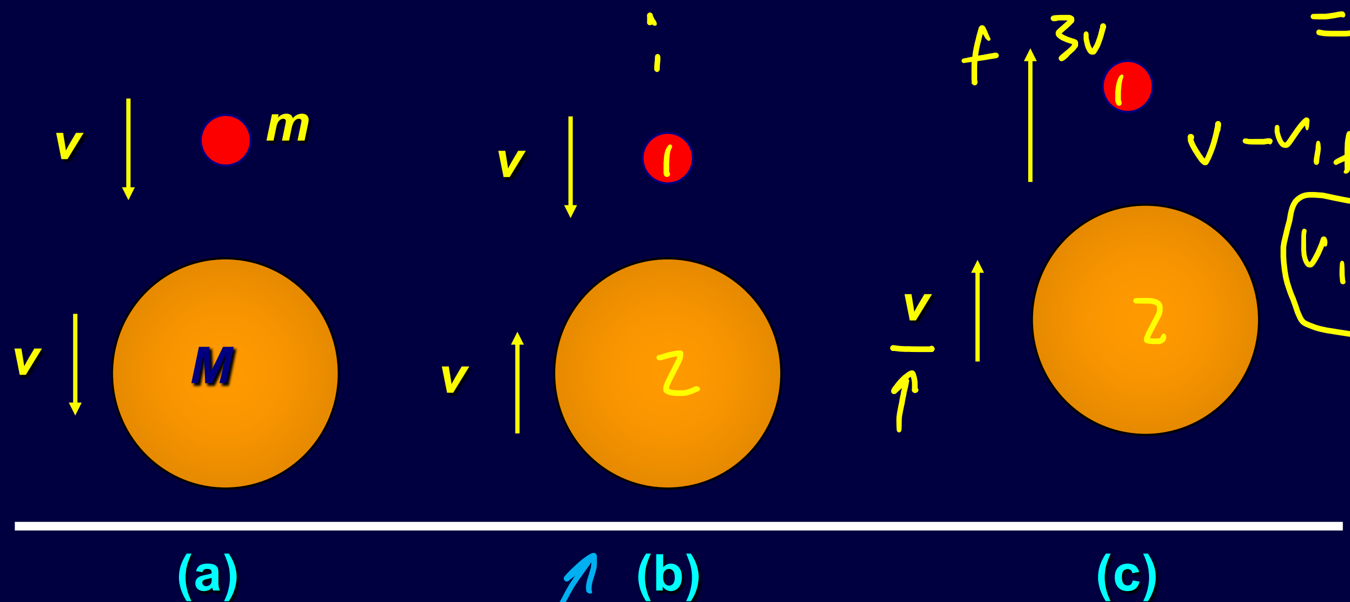
(1) + (3)

$$v_{2f} - v_{1f} = -\left(v_{2i} - v_{1i}\right) \qquad (3)$$

- Together with equation (1) you can solve for just about any 1-d elastic collision

Demo

+y



Elastic

$v_{2f} - v_{1f} = -(v_{2i} - v_{1i})$

$= -(v - (-v))$

$= -2v$

$v - v_{1f} = -2v$

$\boxed{v_{1f} = 3v}$

Elastic Collision

- During an elastic collision both momentum and mechanical energy are conserved:

$$m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + m_2 v_{2i} \qquad (1)$$

$$\tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2 = \tfrac{1}{2} m_1 v_{1i}^2 + \tfrac{1}{2} m_2 v_{2i}^2 \qquad (2)$$

- Combining equation (1) and (2) we get that "speed of approach equals speed of recession":

$$\boxed{v_{2f} - v_{1f} = -\left(v_{2i} - v_{1i}\right)} \qquad (3)$$

- Together with equation (1) you can solve for just about any 1-d elastic collision

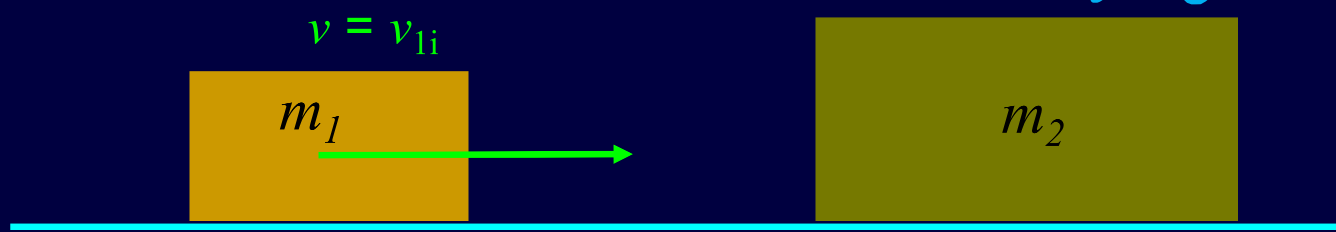
Elastic Collision Example

(1) mom

(3) KE

Consider an elastic head-on collision between a moving object (mass = m_1 and velocity = v_{1i}) and a second object (mass = m_2) initially at rest. Find expressions for the final velocities of these objects after the collision.

$v_{2i} = 0$ ✓



$P_f^x = P_i^x$

$m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i}$

$m_1 v_{1f} + m_2 (v_{1i} + v_{1f}) = m_1 v_{1i}$

$m_1 v_{1f} + m_2 v_{1i} + m_2 v_{1f} = m_1 v_{1i}$

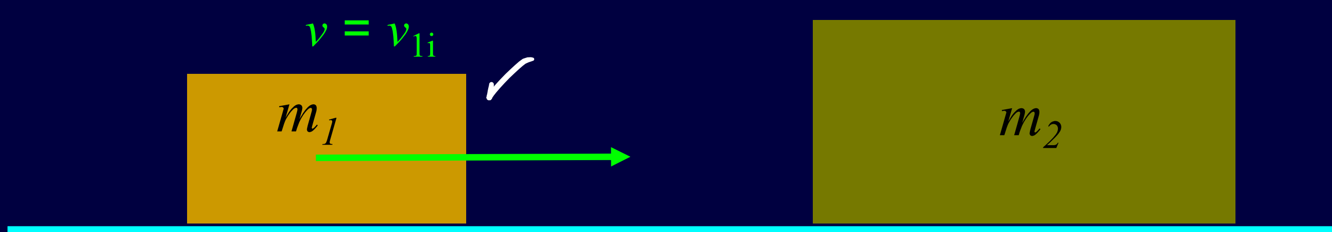
Mech energy conservation

$v_{2f} - v_{1f} = -(\cancel{v_{2i}} - v_{1i})$

$v_{2f} - v_{1f} = v_{1i}$

$\boxed{v_{2f} = v_{1i} + v_{1f}}$

Elastic Collision Example



$$m_1 v_{1f} + m_2 v_{1i} + m_2 v_{1f} = m_1 v_{1i}$$

$$(m_1 + m_2) v_{1f} = (m_1 - m_2) v_{1i}$$

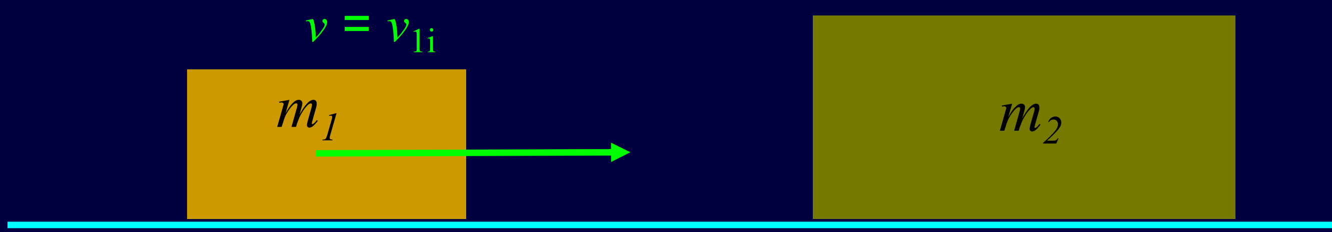
$$\boxed{v_{1f} = \frac{(m_1 - m_2)}{m_1 + m_2} v_{1i}}$$

$$v_{2f} = v_{1f} + v_{1i}$$

$$= \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \left(\frac{m_1 + m_2}{m_1 + m_2}\right) v_{1i}$$

$$\boxed{v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}}$$

Elastic Collision Example



Clicker Question 4:

For the case $m_1 >> m_2$ what are the approximate values of v_{1f} and v_{2f}?

a) $v_{1f} \approx 0$ and $v_{2f} \approx v_{1i}$

b) $v_{1f} \approx v_{1i}$ and $v_{2f} \approx 2v_{1i}$

c) $v_{1f} \approx v_{1i}$ and $v_{2f} \approx 0$

d) Depends on v_{1i}

e) None of the above

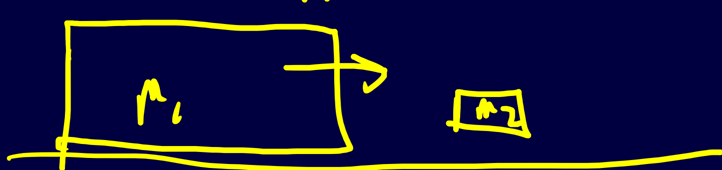
$(V_2: -V_{1i})$

$(0 \quad -V_{1i})$

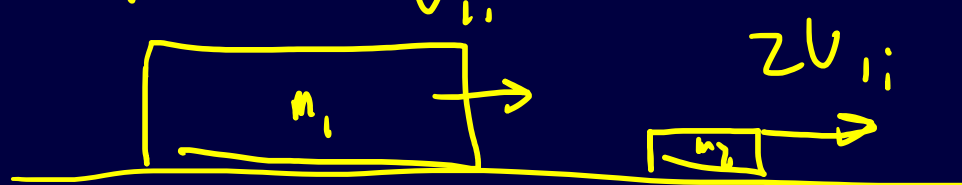
$$v_{1f} = \frac{m_1 - \cancel{m_2}}{m_1 + \cancel{m_2}} v_{1i} \cong V_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + \cancel{m_2}} v_{1i} \cong 2v_{1i}$$

V_{1i}



f ✓



Clicker Question 5:

A bullet with mass 20 grams and velocity 100 m/s collides with a wooden block of mass 2 kg. The wooden block is initially at rest, and is connected to a spring with k = 800 N/m. The other end of the spring is attached to a blue immovable wall. As the bullet imbeds itself into the block we can assume the spring does not have time to compress. As the bullet collides with the block, what principle might be useful?

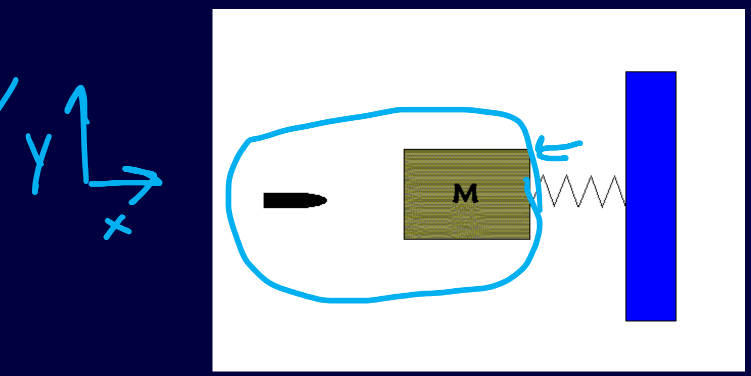
no external forces

(a) Mechanical Energy Conservation

(b) Linear Momentum Conservation ✓

(c) Both (a) and (b)

(d) Neither (a) or (b)



Student Comments

- Correct: Only momentum is conserved because this is an inelastic collision. The bullet imbeds itself in the block, thus dissipating kinetic energy as thermal energy. Since the spring is not compressed during the collision, there are no outside forces so linear momentum is conserved.
- Correct: Energy is not conserved because the friction between the wood and the bullet makes the bullet stop, thus mechanical energy is lost and not conserved. Linear momentum is conserved because the friction is within the block bullet system thus there are no other forces acting on it since the spring doesn't have time to compress.

Question: Problem 2 says that we can assume that the spring does not compress. What is the significance in this?

Clicker Question 6:

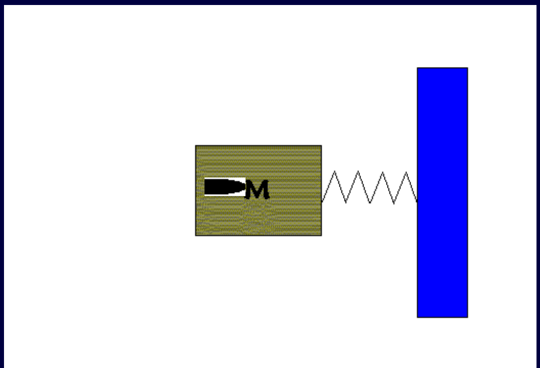
A bullet with mass 20 grams and velocity 100 m/s collides with a wooden block of mass 2 kg. The wooden block is initially at rest, and is connected to a spring with k = 800 N/m. The other end of the spring is attached to an immovable wall. After the bullet is imbedded in the block, what principle might be useful?

(a) Mechanical Energy Conservation

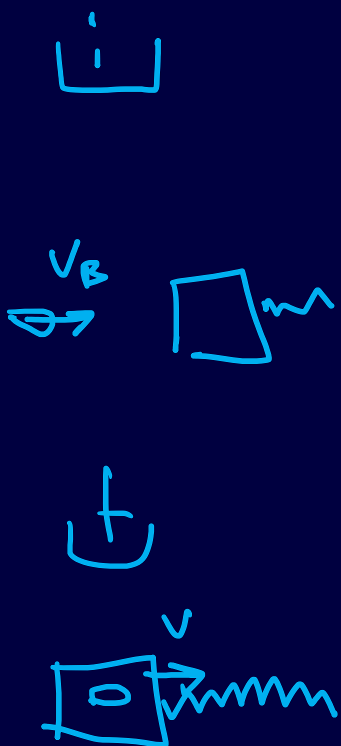
(b) Linear Momentum Conservation

(c) Both (a) and (b)

(d) Neither (a) or (b)



Clicker Question 7:



A bullet with mass 20 grams and velocity 100 m/s collides with a wooden block of mass 2 kg. The wooden block is initially at rest, and is connected to a spring with k = 800 N/m. The other end of the spring is attached to an immovable wall. What is the speed of the block right after the bullet collides with it?

(a) 1 m/s

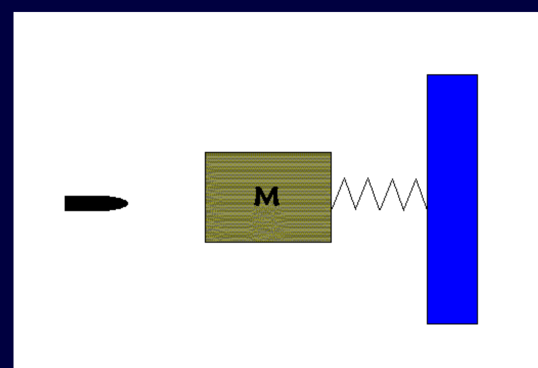
(b) 3 m/s

(c) 4 m/s

(d) 6 m/s

(e) 7 m/s

y ↑ → x



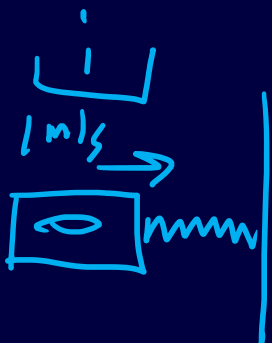
$$p_f^x = p_i^x$$

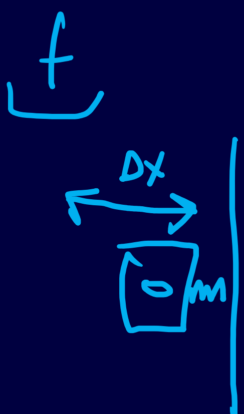
$$m_{TOT} V = m_B v_B$$

$$V = \frac{M_B v_B}{M_{TOT}}$$

$$1\,m/s = \frac{(0.020\,kg)\,100\,m/s}{2.020\,kg}$$

Clicker Question 8:





$\cancel{mgh}$

A bullet with mass 20 grams and velocity 100 m/s collides with a wooden block of mass 2 kg. The wooden block is initially at rest, and is connected to a spring with k = 800 N/m. The other end of the spring is attached to an immovable wall. What is the maximum compression of the spring?

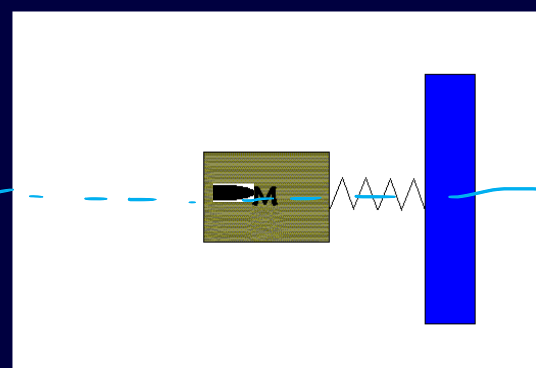
(a) 0.05 m ✓

(b) 0.10 m

(c) 0.13 m

(d) 0.20 m

(e) 0.21 m



$h = 0$

$E_f = E_i$

$\frac{1}{2}kx^2 = \frac{1}{2}m_{TOT}v^2$

$x = \sqrt{\frac{m_{TOT}v^2}{k}} = \sqrt{\frac{(2.02\,kg)(1\,m/s)^2}{800\,N/m}}$

$= 0.05\,m$

Clicker Question 9:

A rifle bullet of mass m = 0.03 kg traveling at v_b = 240 m/s collides with and embeds itself in a pendulum of mass M = 2.88 kg, initially at rest and suspended vertically by massless strings of length L = 2 m. As the bullet collides with the block, what principle might be useful?

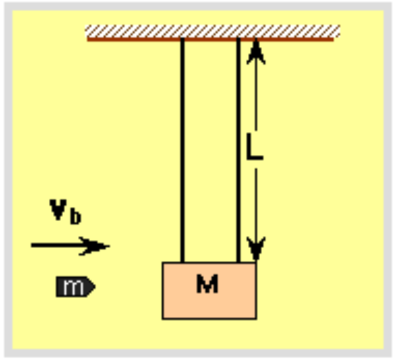
(The bullet/block collision is so fast the pendulum does not have time to swing up until after the collision)

(a) Mechanical Energy Conservation

(b) Linear Momentum Conservation

(c) Both (a) and (b)

(d) Neither (a) or (b)



Clicker Question 10:

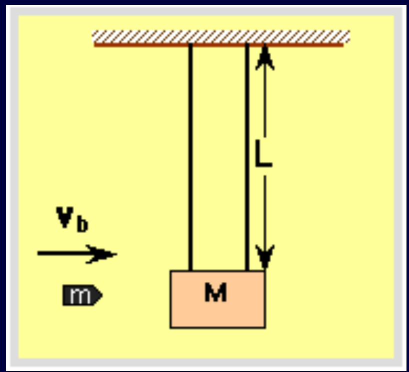
A rifle bullet of mass m = 0.03 kg traveling at v_b = 240 m/s collides with and embeds itself in a pendulum of mass M = 2.88 kg, initially at rest and suspended vertically by massless strings of length L = 2 m. After the bullet is imbedded in the block and the pendulum begins to swing, what principle might be useful?

(a) Mechanical Energy Conservation

(b) Linear Momentum Conservation

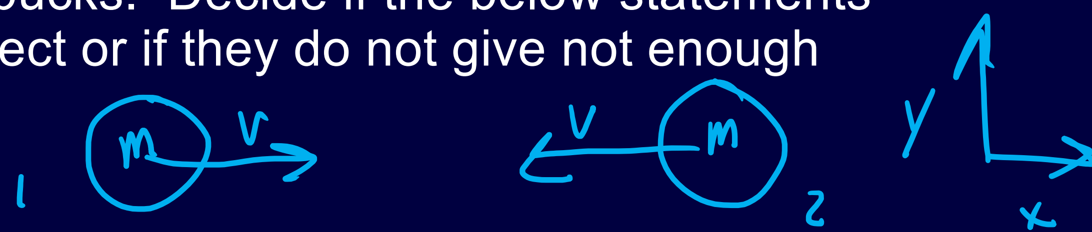
(c) Both (a) and (b)

(d) Neither (a) or (b)



Clicker Question 11:

Two identical pucks move toward each other with equal speeds on a frictionless air track. Assume no external forces act on the pucks. Decide if the below statements are correct, incorrect or if they do not give not enough information.



The total momentum of the system consisting of the two pucks is zero.

$$\Sigma p = +mv - mv$$
$$= 0$$

(a) True

(b) False

(c) Not enough info

Clicker Question 12:

Two identical pucks move toward each other with equal speeds on a frictionless air track. Assume no external forces act on the pucks. Decide if the below statements are correct, incorrect or if they do not give not enough information.

$K = \frac{1}{2} m v^2$

The total kinetic energy of the two pucks is zero.

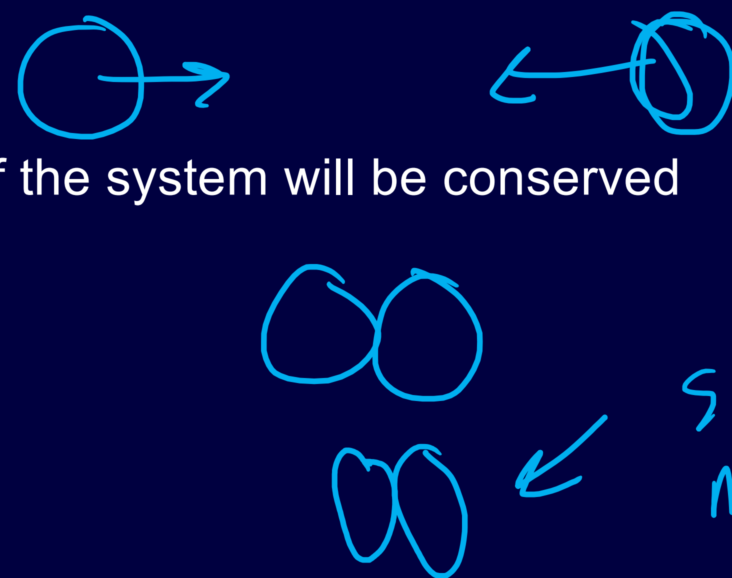
(a) True

(b) False

(c) Not enough info

Clicker Question 14:

Two identical pucks move toward each other with equal speeds on a frictionless air track. Assume no external forces act on the pucks. Decide if the below statements are correct, incorrect or if they do not give not enough information.



The mechanical energy of the system will be conserved during the collision.

(a) True

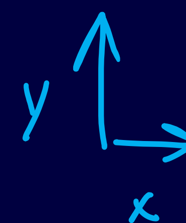
(b) False

(c) Not enough info

Store mechanical energy

Clicker Question 15:

Two identical pucks move toward each other with equal speeds on a frictionless air track. Assume no external forces act on the pucks. Decide if the below statements are correct, incorrect or if they do not give not enough information.



After the two pucks collide the total momentum will not necessarily be the same as before the collision.

internal force

(a) True

(b) False

(c) Not enough info