Physics 2A: Lecture 14

Today's Agenda

- Rotational dynamics
  - Kinetic Energy
  - Moment of Inertia
  - Torque
  - Newton's Second Law for Rotations

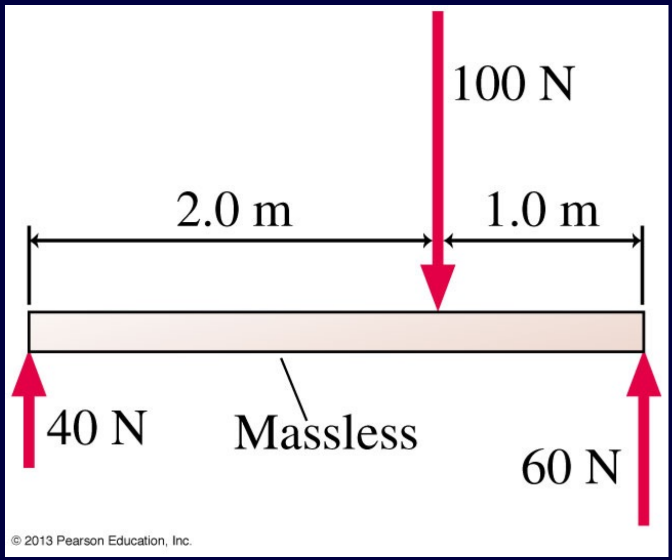
Start Recording

Clicker Question 2:

Is this object in equilibrium?

A. Yes
B. No
C. Maybe

.



Rotational kinetic energy

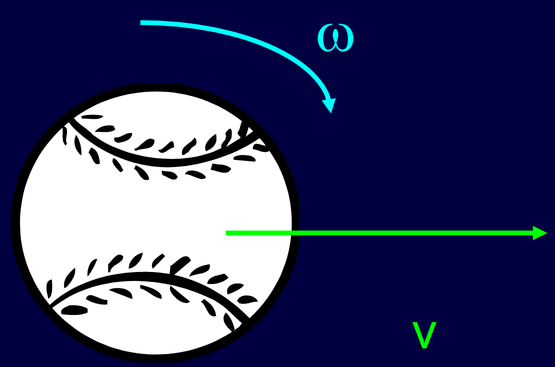
- Kinetic energy
  - Energy of motion
  - $K = \frac{1}{2}mv^2$
- When an object is rotating we also have motion

$$K_{\text{rot}} = \frac{1}{2}I\omega^2$$

  - I is the moment of inertia
  - ω is the angular velocity

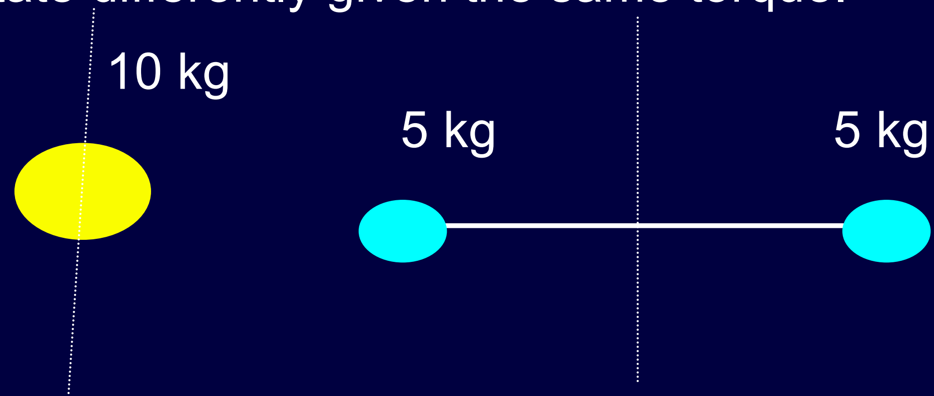
Total kinetic energy for an object

- Now we can represent the total kinetic energy for an object
  - $K_{TOT} = ½mv^2 + ½I\omega^2$
- A baseball usually has both!



Moment of Inertia

- “Rotational ~~version~~ analog of mass”
- How much an object resists an angular acceleration
- How hard it is to rotate an object
- These two objects have the same mass, but will rotate differently given the same torque.



- Moment of Inertia depends on location of axis and distribution of the mass

Will need to calculate moment of inertia for two cases

- First case: Point mass objects
  - An object that is so small compared with its distance from the axis we can forget about its size and consider all of its mass acting at one point
  - $I = \Sigma mr^2$
- Second case: common objects around common rotation axes
  - Textbook pg 300

Clicker Question 3:

What is the moment of inertia for this object about an axis through its center. The length of the rod is 3m and mass of each piece is 5 kg.

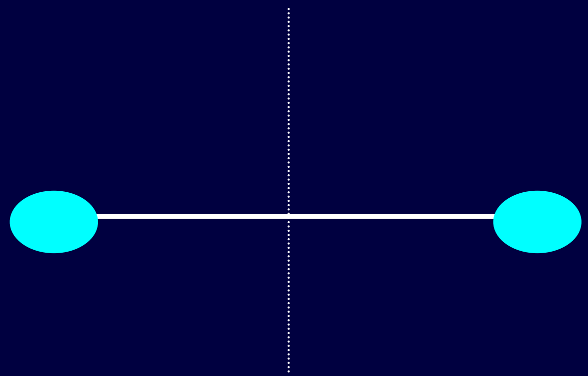
(a) 22.5 kg m^2

(b) 45 kg m^2

(c) 11.25 kg m^2

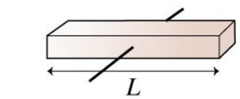
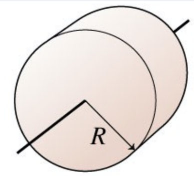
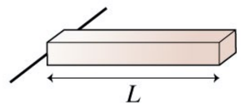
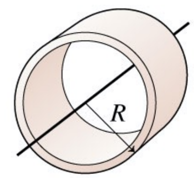
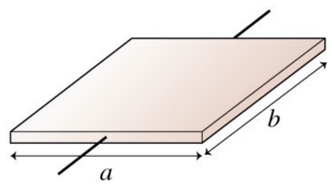
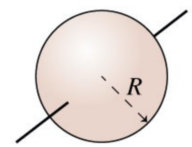
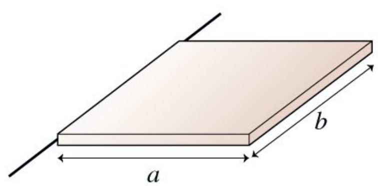
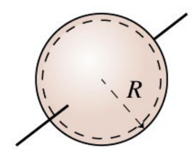
(d) 90 kg m^2

(e) 0 kg m^2



Second Case (pg 300)

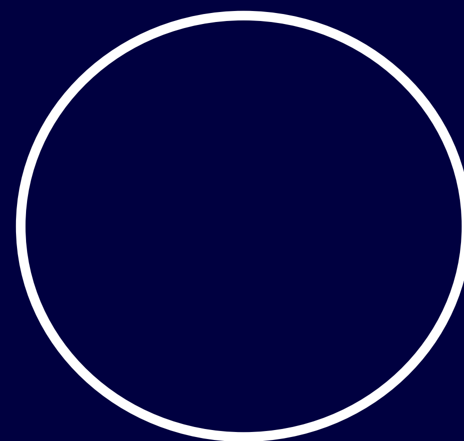
TABLE 12.2 Moments of inertia of objects with uniform density

Object and axis	Picture	I	Object and axis	Picture	I
Thin rod, about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod, about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

You will never need to memorize these!

Second Case (Common objects)

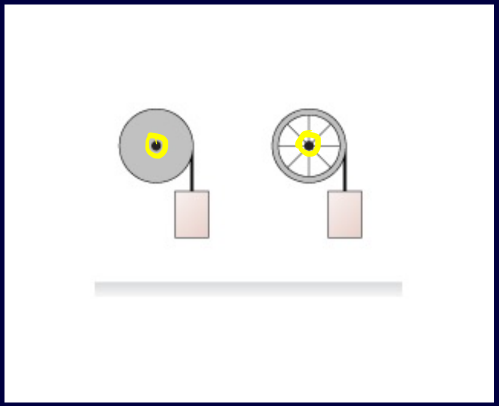
$I = \Sigma m r^2$



Clicker Question 4:

The solid cylinder and cylindrical shell have the same mass, same radius, and turn on frictionless, horizontal axles. A rope is wrapped around each cylinder and tied to a block. The blocks have the same mass and are held the same height above the ground. Both blocks are released simultaneously.

Which block hits the ground first?



a) The one attached to the solid Cylinder

b) The one attached to the cylindrical Shell

c) It's a tie

Clicker Question 4:

The solid cylinder and cylindrical shell have the same mass, same radius, and turn on frictionless, horizontal axles. A rope is wrapped around each cylinder and tied to a block. The blocks have the same mass and are held the same height above the ground. Both blocks are released simultaneously. Which block hits the ground first?

Student comments:

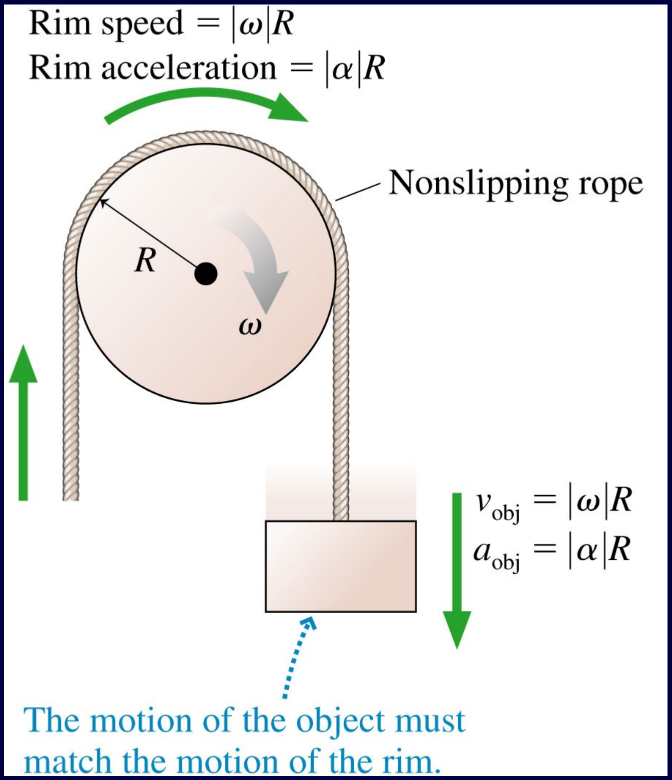
Correct: “The solid cylinder has less moment of inertia compared to the cylindrical shell. Therefore, it will rotate more easily and faster than the cylindrical shell.”

Correct:“The cylindrical shell will be harder to get moving because it has a greater moment of inertia, therefore it will have a lower rotational acceleration than the solid cylinder.”

Constraints Due to Ropes and Pulleys

- A rope passes over a pulley and is connected to an object in linear motion.
- The rope does not slip as the pulley rotates.
- *Tangential* velocity and acceleration of the rim of the pulley must match the motion of the object:

$$v_{obj} = |\omega| R \qquad \text{(motion constraints for a nonslipping rope)}$$
$$a_{obj} = |\alpha| R$$



Question: In problem 1, how come the string does not slip on the frictionless pulley.

Clicker Question 5:

A 15 kg uniform disk of radius R = 0.25 m has a string wrapped around it, and weight is hanging on the string with m = 4.1 kg. The system of the weight and disk is released from rest. When the 4.1 kg weight is moving with a speed of 1.5 m/s, what is the angular speed of the disc?

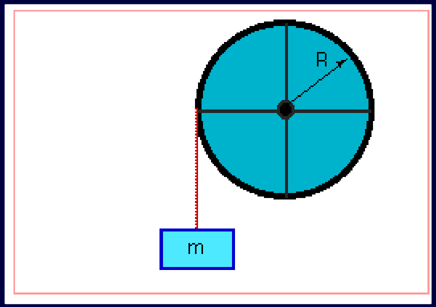
(a) 0.375 rad/s

(b) 0.5 rad/s

(c) 1.5 rad/s

(d) 6 rad/s

(e) 8 rad/s



Clicker Question 6:

A 15 kg uniform disk of radius R = 0.25 m has a string wrapped around it connected to a mass with m = 4.1 kg. The system of the weight and disk is released from rest. At some time later, the 4.1 kg weight is moving with a speed of 1.5 m/s. If the system started at rest, how far has the weight fallen in this time?

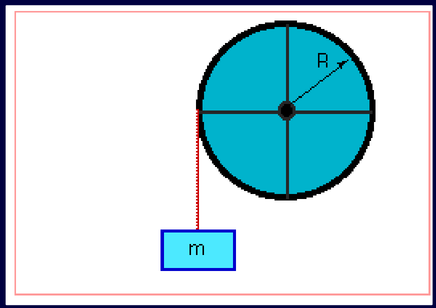
(a) 1.2 m

(b) 0.5 m

(c) 0.3 m

(d) 0.9 m

(e) 3.17 m



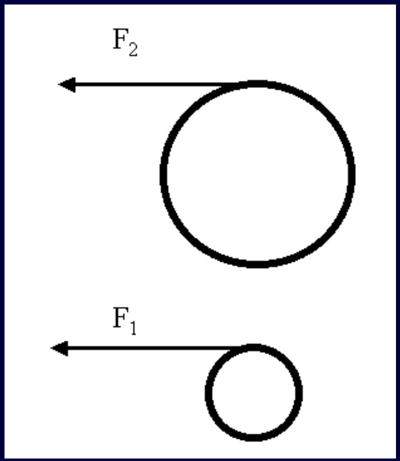
Example

Newton's second law for rotations

- Sum of the torques will equal the moment of inertia times the angular acceleration
- $\Sigma\tau = I\alpha$
- An unbalanced net torque will result in an angular acceleration

Clicker Question 7:

Two wheels can rotate freely about axles through their centers. The wheels have the same mass, but one has twice the radius of the other. Forces F_1 and F_2 are applied as shown. What is F_2/F_1 if the angular acceleration of the wheels is the same?



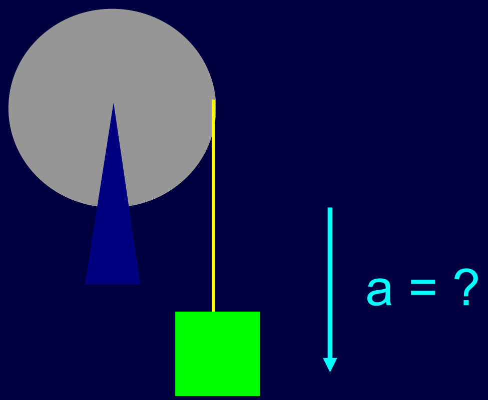
(a) 1

(b) 2

(c) 4

Disk and box

- A box of mass 40 kg is tied to a light string wound around a wheel that has a mass of 30 kg and radius 0.5 m. Find the acceleration of the box.



Clicker Question 8:

A box of m_B = 40 kg is tied to a light string wound around a wheel that has a m_W = 30 kg and radius 0.5 m. Which is the correct equation of motion for the box?

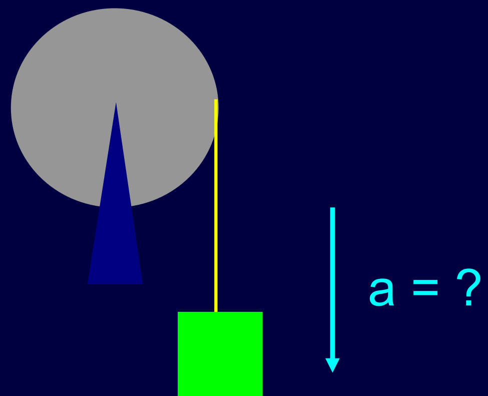
(a) $T = m_B\,g + m_B\,a$

(b) $T = m_B\,g - m_B\,a$

(c) $T = m_B\,g$

(d) $T = m_W\,g - m_W\,a$

(e) $T = m_B\,g - m_W\,a$



Clicker Question 9:

A box of m_B = 40 kg is tied to a light string wound around a wheel that has a m_W = 30 kg and radius 0.5 m. Which is a correct equation of motion for the wheel?

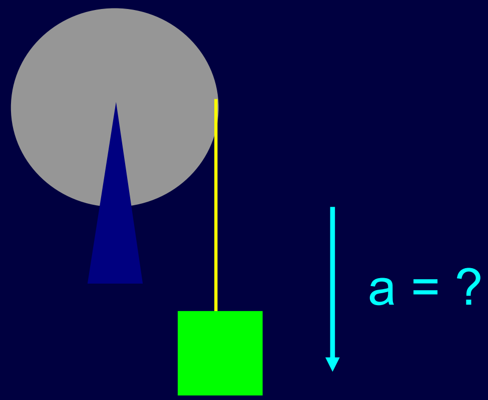
(a) $TR = m_W R^2 \alpha$

(b) $2TR = (1/2) m_W R^2 \alpha$

(c) $TR = (1/2) m_W 4R^2 \alpha$

(d) $TR = (3/4) m_W R^2 \alpha$

(e) $T = (1/2) m_W R \alpha$



Clicker Question 10:

A box of m_B = 40 kg is tied to a light string wound around a wheel that has a m_W = 30 kg and radius 0.5 m. How do the angular acceleration α of the wheel and the linear acceleration a of the box compare?

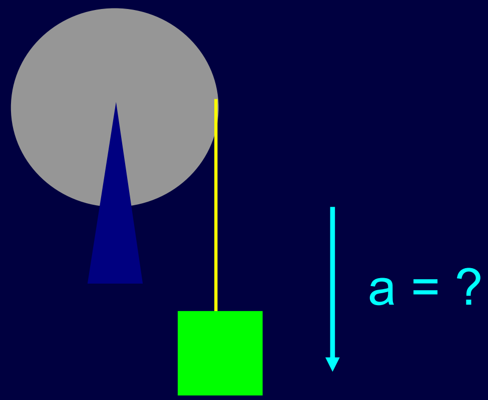
(a) $a = \alpha R$

(b) $a = \alpha 2R$

(c) $a = \alpha 4R$

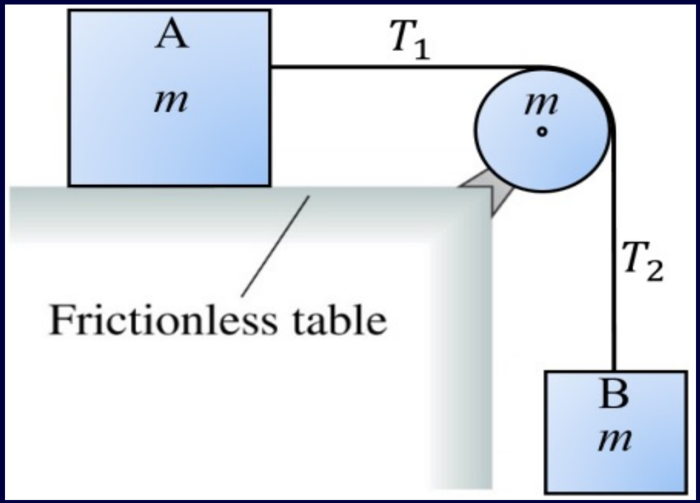
(d) $a = \alpha R/2$

(e) $a = \alpha R/4$



Clicker Question 11:

- Block A slides upon a frictionless table, as shown. It is connected to a string, the other end of which is connected to a hanging block B. The string goes over a frictionless, massive pulley. As the two blocks move, the string does not slip on the pulley. At the moment shown in the figure, which of the following statements is true about the tension in the top part of the string, T_1, the tension in the lower part of the string, T_2, and the force of gravity on block B, mg?



(a) $T_1 = T_2 = mg$

(b) $T_1 = T_2 < mg$

(c) $T_1 < T_2 = mg$

(d) $T_1 = T_2 > mg$

(e) $T_1 < T_2 < mg$

Clicker Question 11: