Physics 2A: Lecture 14

Today's Agenda

- Rotational dynamics
 - Kinetic Energy
 - Moment of Inertia
 - Torque
 - Newton's Second Law for Rotations

Clicker Question 2:

Is this object in equilibrium?



Rotational kinetic energy

- Kinetic energy
 - Energy of motion
 - K = $\frac{1}{2}mv^2$

$$\begin{array}{ccc} \mathfrak{M} & \longrightarrow & \widetilde{L} \\ \mathcal{V} & \longrightarrow & \mathcal{W} \end{array}$$

When an object is rotating we also have motion

$$K_{\rm rot} = \frac{1}{2}I\omega^2$$

I is the moment of inertia

• ω is the angular velocity

Total kinetic energy for an object

- Now we can represent the total kinetic energy for an object
 - $K_{TOT} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$
- A baseball usually has both!



Moment of Inertia



- "Rotational management of mass"
- How much an object resists an angular acceleration
- How hard it is to rotate an object¹
- These two objects have the same mass, but will rotate differently given the same torque.



distribution of the mass

Will need to calculate moment of inertia for two cases

- First case: Point mass objects
 - An object that is so small compared with its distance from the axis we can forget about its size and consider all of its mass acting at one point

$$I = \Sigma mr^2 = M_1 \Gamma_1 + M_2 \Gamma_2 + M_3 \Gamma_3 + \dots$$

Tamr

Second case: common objects around common rotation axes

Textbook pg 300

Clicker Question 3:

What is the moment of inertia for this object about an axis through its center. The length of the rod is 3m and mass of each piece is 5 kg.

(a) 22.5 kg m²
(b) 45 kg m²
(c) 11.25 kg m²
(d) 90 kg m²
(e) 0 kg m²



Ground Cont

Clicker Question 3:

- Find moment of inertia for this object, length of rod is 3m and mass of each piece is 5 kg.
- $I = \Sigma m_i r_i^2 = 5kg(1.5 m)^2 + 5kg(1.5 m)^2 = 22.5 kg m^2$
- Bigger mass, more inertia
- Mass farther from axis of rotation, more inertia
- Always add moments!





Second Case (pg 300)

ML



You will never need to memorize these!



Clicker Question 4:

The solid cylinder and cylindrical shell have the same mass, same radius, and turn on frictionless, horizontal axles. A rope is wrapped around each cylinder and tied to a block. The blocks have the same mass and are held the same height above the ground. Both blocks are released simultaneously.

Which block hits the ground first?

- a) The one attached to the solid Cylinder
- b) The one attached to the cylindrical Shell
- c) It's a tie

Clicker Question 4: The solid cylinder and cylindrical shell have the same mass, same radius, and turn on frictionless, horizontal axles. A rope is wrapped around each cylinder and tied to a block. The blocks have the same mass and are held the same height above the ground. Both blocks are released simultaneously. Which block hits the ground first?

Student comments:

Correct: "The solid cylinder has less moment of inertia compared to the cylindrical shell. Therefore, it will rotate more easily and faster than the cylindrical shell."

Correct: "The cylindrical shell will be harder to get moving because it has a greater moment of inertia, therefore it will have a lower rotational acceleration than the solid cylinder."

Constraints Due to Ropes and Pulleys

- A rope passes over a pulley and is connected to an object in linear motion.
- motion.
 The rope does not slip as the pulley rotates.
- Tangential velocity and acceleration of the rim of the pulley must match the motion of the object:





(motion constraints for a nonslipping rope)

Question: In problem 1, how come the string does not slip on the frictionless pulley.

Clicker Question 5:

- A 15 kg uniform disk of radius R = 0.25 m has a string wrapped around it, and weight is hanging on the string with m = 4.1 kg. The system of the weight and disk is released from rest. When the 4.1 kg weight is moving with a speed of 1.5 m/s, what is the angular speed of the disc?
- (a) 0.375 rad/s
- (b) 0.5 rad/s
- (c) 1.5 rad/s
- (d) 6 rad/s
- (e) 8 rad/s





」 え M R² Clicker Question 6:

- A 15 kg uniform disk of radius R = 0.25 m has a string wrapped around it connected to a mass with m = 4.1 kg. The system of the weight and disk is released from rest. At some time later, the 4.1 kg weight is moving with a speed of 1.5 m/s. If the system started at rest, how far has the weight fallen in this time?
- (a) 1.2 m
- (b) 0.5 m
- (c) 0.3 m
- (d) 0.9 m
- (e) 3.17 m



 $m_{i} = 4,145$ $m_{z} = 1545$ $V = 1.5 ml_{s}$ $g = -9,81 ml_{s}^{2}$





• An unbalanced net torque will result in an angular acceleration

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Clicker Question 7:

Two wheels can rotate freely about axles through their centers. The wheels have the same mass, but one has twice the radius of the other. Forces F_1 and F_2 are applied as shown. What is F_2/F_1 if the angular acceleration of the wheels is the same?

(a) 1
(b) 2
(c) 4

I=ZmR2



F = F R**Clicker Question 7:**

Two wheels can rotate freely about axles through their centers. The wheels have the same mass, but one has twice the radius of the other. Forces F_1 and F_2 are applied as shown. What is F_2/F_1 if the angular acceleration of the wheels is the same?

 $\frac{F_2}{F_1} = 2$

 $\Sigma \tau_2 = I_2 \alpha$

 $F_2(2R) = \frac{1}{2}m(2R)^2 \alpha \qquad \frac{F_2}{R} = \frac{1}{2}m^2 \frac{1}{2}m^2$ F_1

 $F_2 = \frac{1}{2}m(2R) \alpha$

- $\Sigma \tau_1 = I_1 \alpha$
- $F_1(R) = \frac{1}{2}m(R)^2 \alpha$

 $F_1 = \frac{1}{2}m(R) \alpha$



Disk and box

 A box of mass 40 kg is tied to a light string wound around a wheel that has a mass of 30 kg and radius 0.5 m. Find the acceleration of the box.





Clicker Question 8:

Let's use Newton's second law on the box

- $\Sigma F = m_B a$
- $m_B g T = m_B a$ $- T = m_B g - m_B a$



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Clicker Question 9:

A box of $m_B = 40$ kg is tied to a light string wound around a wheel that has a $m_W = 30$ kg and radius 0.5 m. Which is a correct equation of motion for the wheel?

(a) TR =
$$m_W R^2 \alpha$$

(b)
$$2TR = (1/2)m_W R^2 \alpha$$

(c) TR =
$$(1/2)m_W 4R^2 \alpha$$

(d) TR =
$$(3/4)m_W R^2 \alpha$$

(e)
$$T = (1/2)m_W R\alpha$$



Question: I don't understand why we sometimes consider the moment of inertia as I=1/2mr^2 and in other situations as I=mr^2.

Clicker Question 9:

 $T = \frac{1}{2} M \omega R'$

- The string will pull with a tension T at a distance of R from the axis of rotation for the disk.
- so : τ =TR
- That is the only torque
- So $\Sigma \tau = I\alpha$
- TR = Iα
- $I = (1/2)m_W R^2$
- TR = $(1/2)m_W R^2 \alpha$
- T =(1/2)m_wRα



Clicker Question 10:

- A box of $m_B = 40$ kg is tied to a light string wound around a wheel that has a $m_W = 30$ kg and radius 0.5 m. How do the angular acceleration α of the wheel and the linear acceleration a of the box compare?
- (a) $a = \alpha R$ (b) $a = \alpha 2R$ (c) $a = \alpha 4R$
- (d) $a = \alpha R/2$
- (e) $a = \alpha R/4$



Q
Q
Disk and box

$$A = \lambda R$$
 • Now we remember $a_T = R\alpha$
• So we can relate α and a
 $\lambda = \frac{\alpha}{R}$ • $T = (1/2)m_WR \alpha$
 $V \cdot T = m_Bg \cdot m_Ba$
 $M_B f - M_B q = \frac{1}{2}m_W R d$
 $= \frac{1}{2}m_W R q = \frac{1}{2}m_W q$
 $m_B f = m_B q + \frac{1}{2}m_W q$
 $m_B f = (m_B + \frac{1}{2}m_W) q$
 $\xi F = m_{PT} q$
 $a = \frac{m_B g}{(\frac{1}{2}m_W + m_B)}$

Clicker Question 11:

Block A slides upon a frictionless table, as shown. It is connected to a string, the other end of which is connected to a hanging block B. The string goes over a frictionless, massive pulley. As the two blocks move, the string does not slip on the pulley. At the moment shown in the figure, which of the following statements is true about the tension in the top part of the string, *T*₁, the tension in the lower part of the string, *T*₂, and the force of gravity on block B, *mg*?

(a)
$$T_1 = T_2 = mg$$

(b) $T_1 = T_2 < mg$
(c) $T_1 < T_2 = mg$
(d) $T_1 < T_2 = mg$
(e) $T_1 < T_2 < mg$



