

# Physics 2A: Lecture 16

## Today's Agenda

CH12



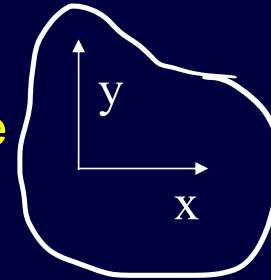
CH15

- Rotational dynamics
  - Torque
  - $\Sigma \tau = I \alpha$
  - Angular Momentum

rolling  
w/o slipping

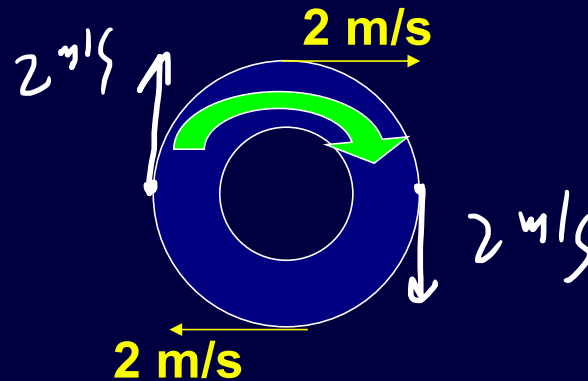
## Rolling

A wheel is spinning clockwise such that the speed of the outer rim is 2 m/s.



What is the velocity of the top of the wheel relative to the ground?  $+2 \text{ m/s}$

What is the velocity of the bottom of the wheel relative to the ground?  $-2 \text{ m/s}$



**Question:** What is an example of rolling without slipping in real life  
I'm having troubles picturing it.

## Rolling *v/o* slipping

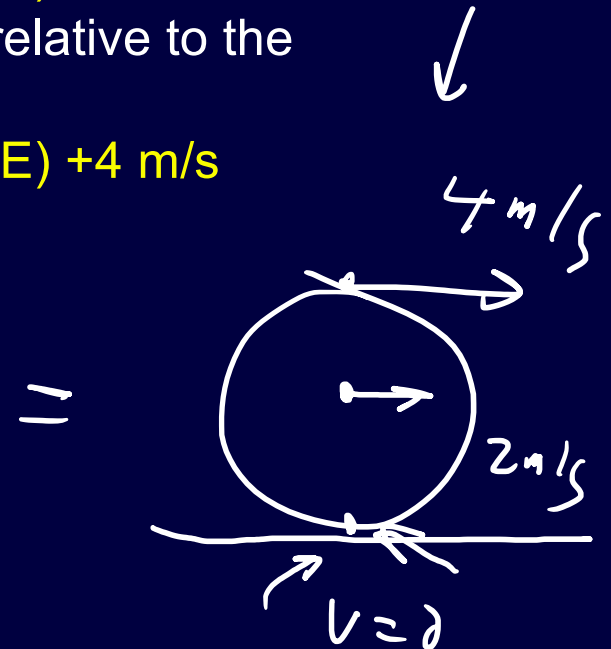
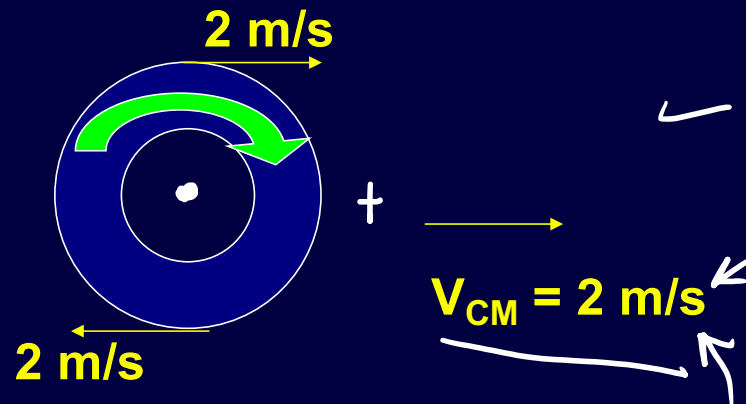
You now carry the spinning wheel to the right at 2 m/s.

What is the velocity of the top of the wheel relative to the ground?

A) -4 m/s   B) -2 m/s   C) 0 m/s   D) +2 m/s   E) +4 m/s

What is the velocity of the bottom of the wheel relative to the ground?

A) -4 m/s   B) -2 m/s   C) 0 m/s   D) +2 m/s   E) +4 m/s



## Rolling without slipping

- The linear velocity is the same as the tangential velocity on the edge of the rolling object
- So we can use the viewer equation
- $v_{CM} = \omega r$

$\frac{1}{2}R$

disk

## Rolling Without Slipping

- When a disk rolls we assume it does not slip and the outside of the ball moves with a velocity such that:  $v = \omega r$

- $KE_{TOT} = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2$

- $\omega = v/r$        $I = \frac{1}{2}mr^2$

disk

$$v_{cm} = \omega R$$

$$\omega = \frac{v_{cm}}{R}$$

$$KE_{TOT} = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$$

$$KE_{TOT} = \frac{1}{2}MV^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\frac{V^2}{R^2} = \frac{1}{2}MV^2 + \frac{1}{4}MV^2$$

$$\rightarrow \boxed{KE = \frac{3}{4}MV^2}$$

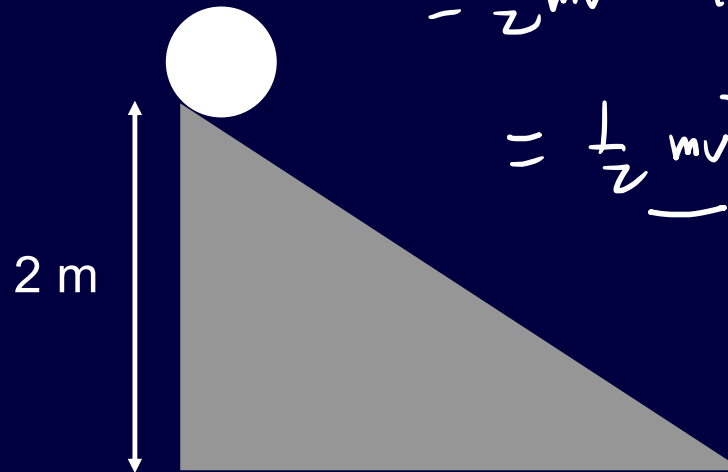
$$v = \omega r$$

## Clicker Question 2:

$$I_{\text{cm}} = m R^2$$

- A hollow cylinder of mass  $m$  rolls down an incline of height 2 m. What velocity will its center of mass have at the bottom of the incline?

- (a) 9.0 m/s
- (b) 7.13 m/s
- (c) 4.42 m/s
- (d) 4.0 m/s
- (e) 5.6 m/s



$$\begin{aligned}
 K &= \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 \\
 &= \frac{1}{2} m v^2 + \frac{1}{2} m R^2 \left[ \frac{v^2}{R^2} \right] \\
 &= \frac{1}{2} m v^2 + \frac{1}{2} m v^2 \\
 &= m v^2
 \end{aligned}$$

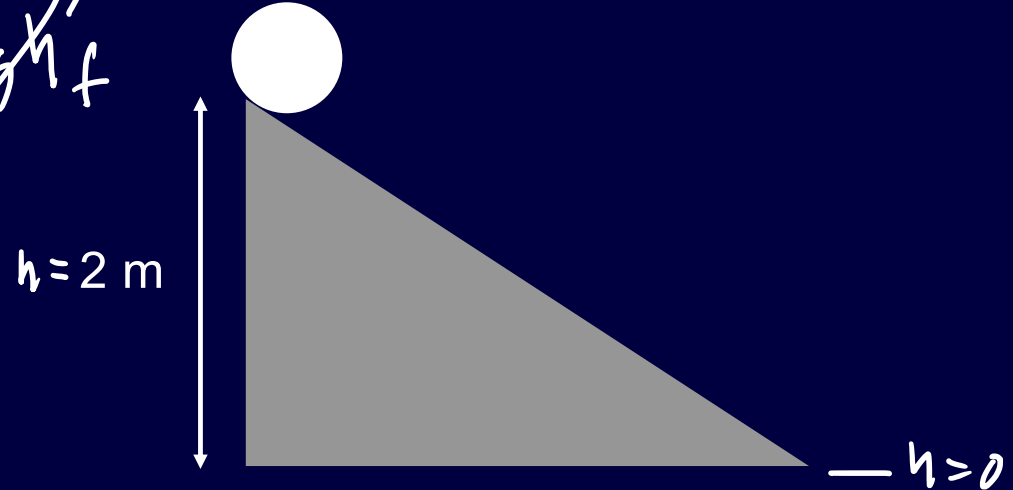
## Clicker Question 2:

- A hollow cylinder of mass  $m$  rolls down an incline of height 2 m. What velocity will its center of mass have at the bottom of the incline?

$$E_i = E_f$$
$$\cancel{K} + mgh = mv^2 + \cancel{mgh_f}$$

$$v = \sqrt{gh}$$

$$= \underline{4.42 \text{ m/s}}$$



### Clicker Question 3:

Consider the following three objects, each of the same mass and radius:

(1) a solid sphere (2) a solid disk (3) a hoop

All three are released from rest at the top of an inclined plane. The three objects proceed down the incline undergoing rolling motion without slipping. In which order do the objects reach the bottom of the incline?

(a) 1, 2, 3

(b) 2, 3, 1

(c) 3, 1, 2

(d) 3, 2, 1

(e) All three reach the bottom at the same time.

$$I_{\text{hoop}} = mr^2$$

$$I_{\text{SS}} = \frac{2}{5} mr^2$$

$$I_{\text{SD}} = \frac{1}{2} mr^2$$

### Clicker Question 3:

$$KE_{TOT} = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$$

$$I = FMR^2$$

$F \equiv$  fraction

$$KE_{TOT} = \frac{1}{2}MV^2 + \frac{1}{2}(FMR^2)\frac{V^2}{R^2} = \frac{1}{2}MV^2 + \frac{F}{2}MV^2$$

$$KE = \frac{(F+1)}{2}MV^2$$

$$SS \Rightarrow \frac{2}{5}$$

$$\rightarrow H \Rightarrow 1$$

$$D \Rightarrow \frac{1}{2}$$

$$\frac{(F+1)}{2}MV^2 = MgH$$

Only depends on the fraction  $F$ !

$$\downarrow V = \sqrt{\frac{2gH}{F+1}} \uparrow$$

$$\vec{p} = m\vec{v}$$

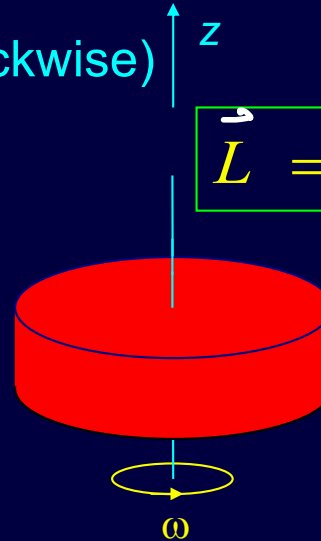
$$\vec{p} = m\vec{v}$$

## Angular momentum of a rigid body about a fixed axis:

- Here we have a disk with moment of inertia  $I$  and angular velocity  $\omega$
- It has angular momentum of  $I\omega$
- Rotation is positive (CounterClockwise)

$$\vec{L} = I\vec{\omega}$$

Angular momentum is a vector!



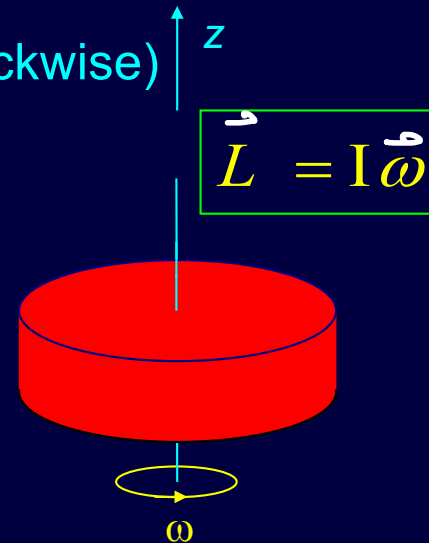
$$\vec{p} = m\vec{v}$$

## Angular momentum of a rigid body about a fixed axis:

- Here we have a disk with moment of inertia  $I$  and angular velocity  $\omega$
- It has angular momentum of  $I\omega$
- Rotation is positive (CounterClockwise)

What makes change in angular motion about a fixed axis difficult?

Angular momentum gives a sense of how hard it is to stop a rotating object



## Linear Momentum

- Moving object has linear momentum such that:

$$\vec{p} = m\vec{v}$$

- Vector quantity in same direction as velocity

$$p_x = mv_x$$

- Units: kg m/s or N s

$$p_y = mv_y$$

What makes change in motion difficult?

Momentum gives a sense of how hard it is to stop an object

Student, “I really don't understand what exactly momentum is in terms of a quantity.”

## Linear and Angular

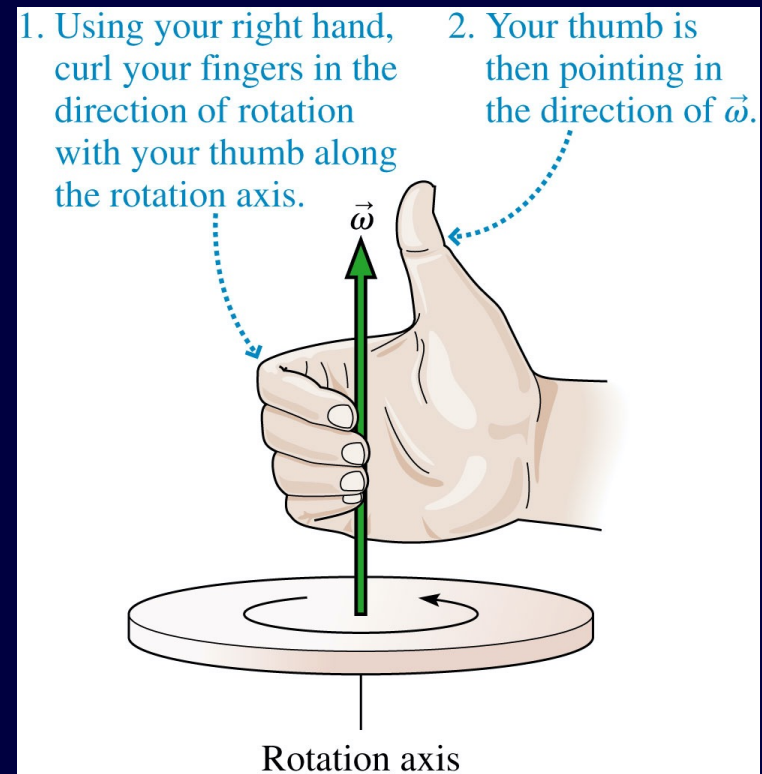
	Linear	Angular
Displacement	$s$	$\theta$
Velocity	$v$ ✓	$\omega$ ✓
Acceleration	$a$	$\alpha$
Inertia	$m$ ✓	$I$ ✓
K	$\frac{1}{2} m v^2$	$\frac{1}{2} I \omega^2$
N2L	$F = ma$	$\tau = I\alpha$
Momentum	$p = mv$	$L = I\omega$

Today

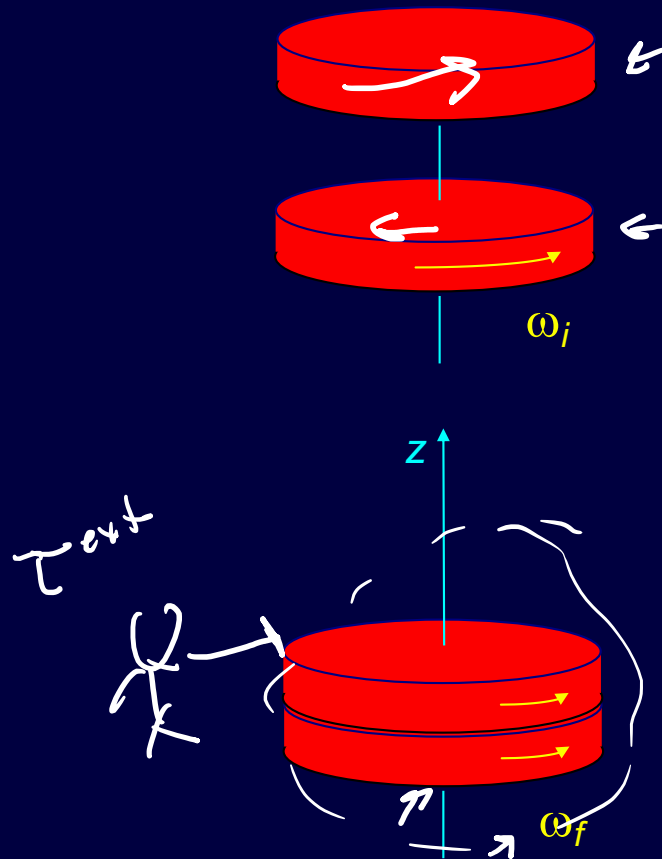
$\omega$ 
 $\leftarrow$ 
 $L = \underline{I \omega}$

# Angular Momentum

- The magnitude of the angular velocity vector is  $\omega$ .
- The angular velocity vector points along the axis of rotation in the direction given by the right-hand rule as illustrated.



## External/Internal Torques

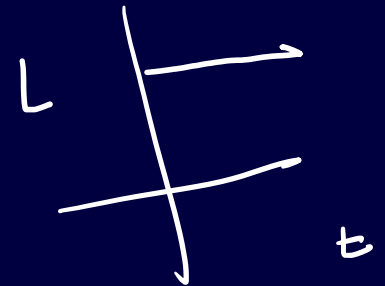


$$\Rightarrow \vec{\tau}_{net} = \frac{d\vec{L}}{dt}$$

$$\Rightarrow \vec{\tau}_{net}^{ext} + \vec{\tau}_{net}^{int} = \frac{d\vec{L}}{dt}$$

$$\vec{\tau}_{net}^{ext} + 0 = \frac{d\vec{L}}{dt}$$

$$\vec{\tau}_{net}^{ext} = \frac{d\vec{L}}{dt}$$



# Conservation of Angular Momentum

- Newton's second law for rotations

$$\vec{\tau}_{\text{ext}} = \frac{d\vec{L}}{dt}$$

$$\vec{L} = I \vec{\omega}$$

- When the external torques on a system sum to zero, the total angular momentum of the system will be conserved
- The angular momentum of an isolated system is conserved

## Conservation of Linear Momentum

- The angular momentum of an isolated system is conserved

$$\vec{L}_f = \vec{L}_i$$

$$\sum I\vec{\omega}_f = \sum I\vec{\omega}_i$$

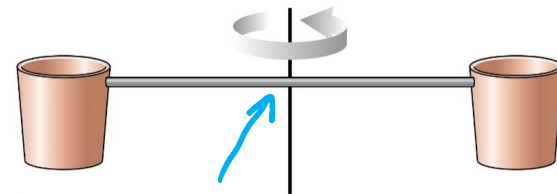
object 1   object 2   object 1   object 2

$$I\omega_f + I\omega_f = I\omega_i + I\omega_i$$

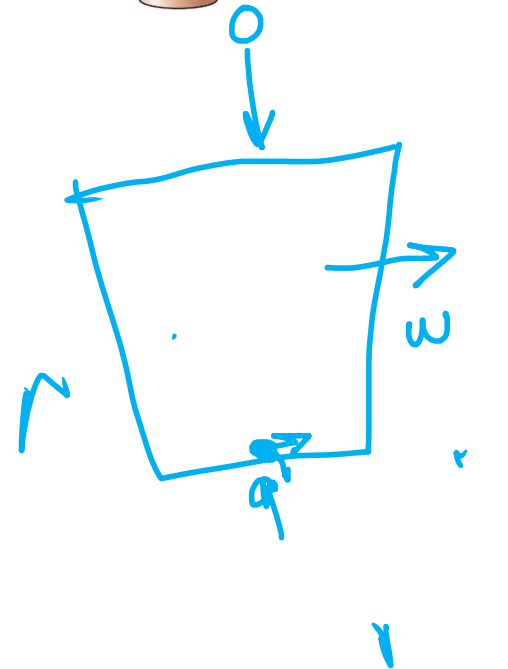
Student: I don't really understand what you mean by external torque.  
External force is easy to understand, but what could be an external torque in a sample situation?

## Clicker 4:

Two buckets spin around in a horizontal circle on frictionless bearings. Suddenly, it starts to rain. As a result,

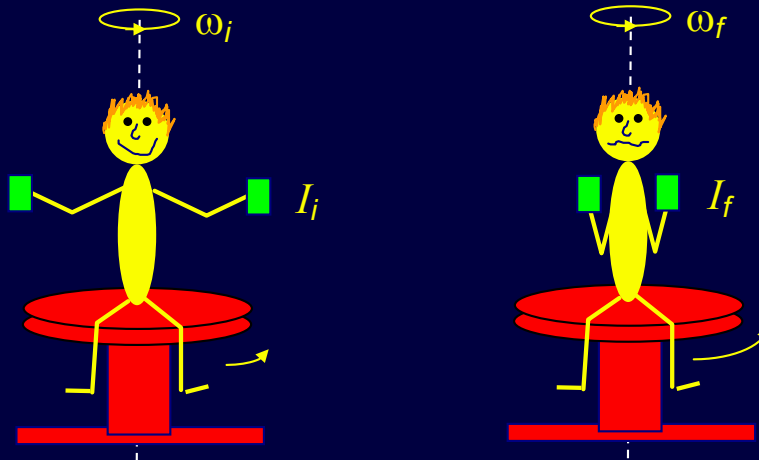


- A. The buckets speed up because the potential energy of the rain is transformed into kinetic energy.
- B. The buckets continue to rotate at constant angular velocity because the rain is falling vertically while the buckets move in a horizontal plane.
- C. The buckets slow down because the angular momentum of the bucket + rain system is conserved.
- D. The buckets continue to rotate at constant angular velocity because the total mechanical energy of the bucket + rain system is conserved.
- E. None of the above.



## Demonstrations: Angular momentum

- What happens to your angular momentum as you pull in your arms? Does it increase, decrease or stay the same? What happens to your angular velocity as you pull in your arms?



## Example: Rotating Table...

There are no external torques acting on the student-stool system, so angular momentum will be conserved.

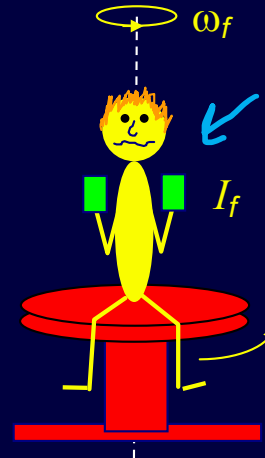
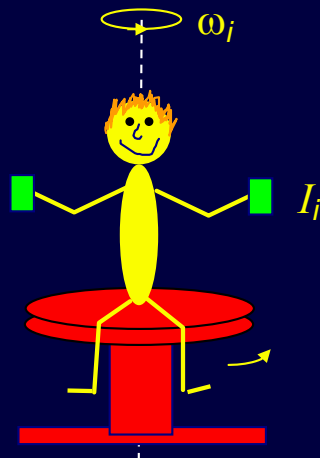
- Initially:  $L_i = I_i \omega_i$
- Finally:  $L_f = I_f \omega_f$



$$\frac{\omega_f}{\omega_i} = \frac{I_i}{I_f} > 1$$

$$I_i \omega_i = I_f \omega_f$$

$$\frac{\omega_f}{\omega_i} = \frac{I_i}{I_f} > 1$$



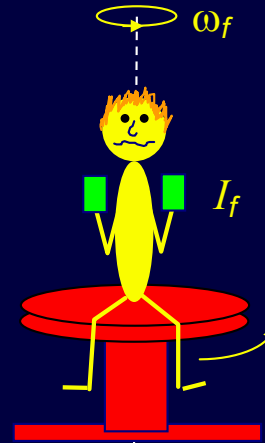
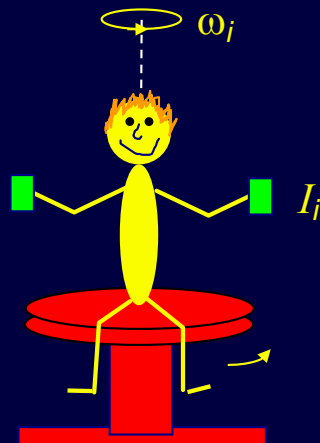
## Clicker Question 5:

What happens to your kinetic energy as you pull in your arms?

- (a) increases
- (b) decreases
- (c) stays the same

No external torques, ang mom is conserved

$$L = I\omega \uparrow$$



$$\uparrow K = \frac{1}{2} \downarrow I \omega^2 \uparrow$$

$$K = \frac{1}{2} I \omega^2 \left( \frac{I}{I} \right)$$

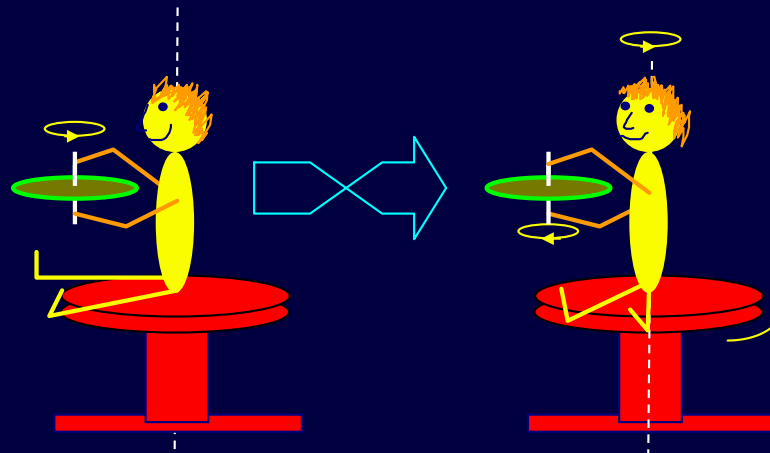
$$\uparrow K = \frac{1}{2} \frac{I^2 \omega^2}{I} = \frac{L^2}{2I} \downarrow$$

## Clicker Question 6:

A student sits on a stool, that is free to rotate on a frictionless axis, holding a bike wheel. The wheel is initially spinning CCW in the horizontal plane (as viewed from above) She now turns the bike wheel over. What happens?

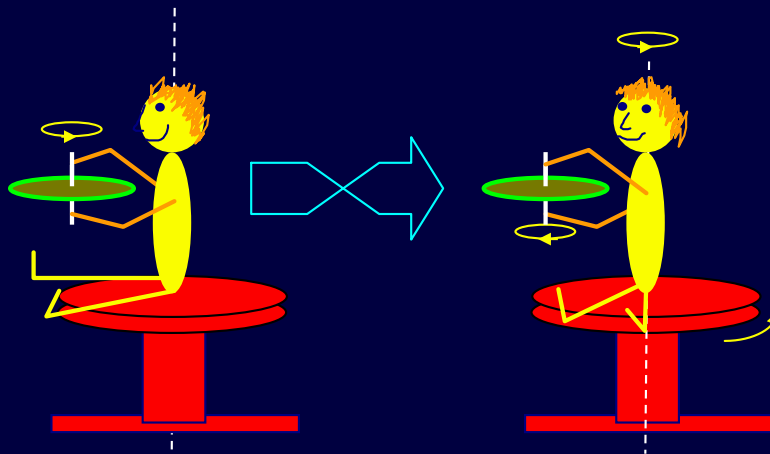
- (a) She starts to spin CCW.
- (b) She starts to spin CW.
- (c) Nothing

No external torques, ang mom is conserved



## Demo: Turning the bike wheel.

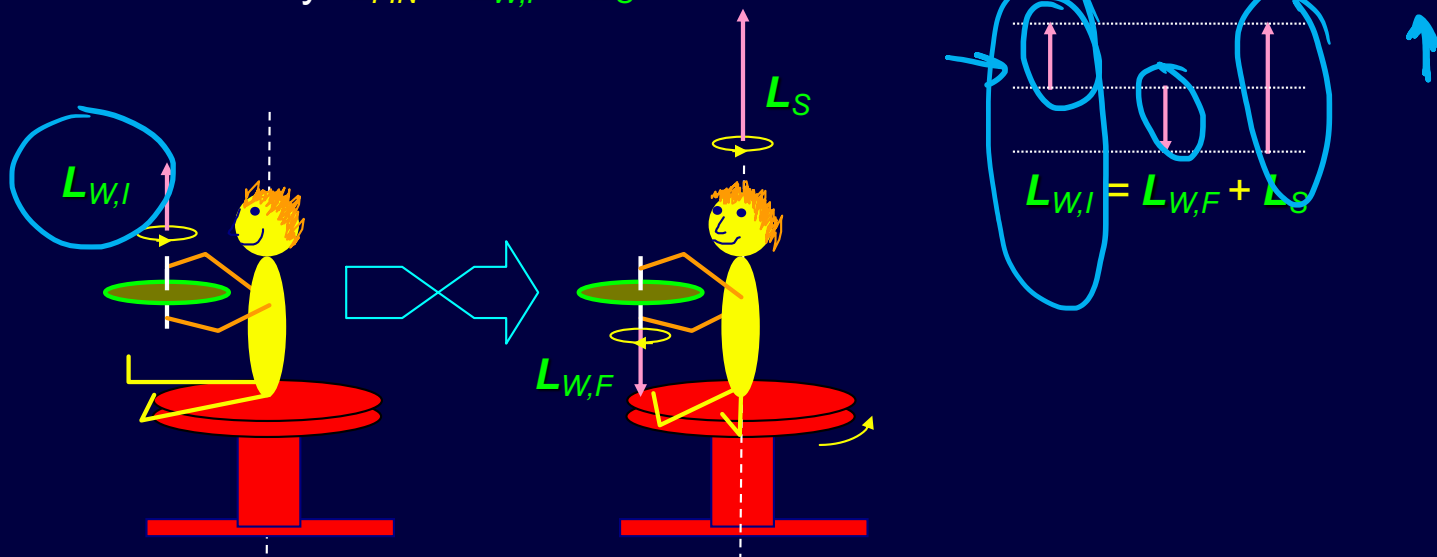
- A student sits on the rotatable stool holding a bicycle wheel that is spinning in the horizontal plane. She flips the rotation axis of the wheel  $180^\circ$ , and finds that she herself starts to rotate.
  - What's going on?



## Turning the bike wheel...

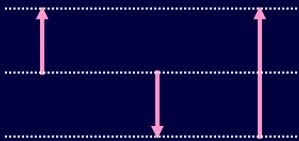
- Since there are no external torques acting on the student-stool system, angular momentum is conserved.
  - Initially:  $L_{INI} = L_{W,I}$
  - Finally:  $L_{FIN} = L_{W,F} + L_S$

CLW +



Student: the spinning bicycle wheel and the merry-go-round are not connected so why would stopping the wheel have any effect on the merry-go-round?

## Turning the bike wheel...

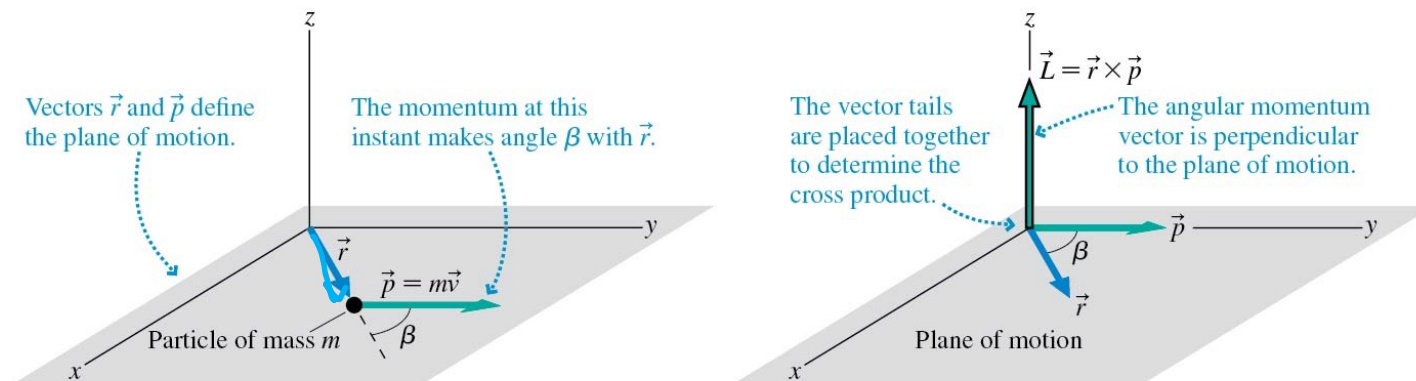


A diagram showing a horizontal rectangle representing a wheel. Three vertical arrows are drawn: one on the left pointing up, one in the center pointing down, and one on the right pointing up. The arrows are positioned between two horizontal dotted lines.

$$L_{W,I} = L_{W,F} + L_S$$

# Angular Momentum of a Particle

- A particle of mass  $m$  is moving. The particle's momentum vector makes an angle  $\beta$  with the position vector.



- We define the particle's angular momentum vector relative to the origin to be

$$\vec{L} \equiv \vec{r} \times \vec{p} = (mrv \sin \beta, \text{direction of right-hand rule})$$

## Rotating Neutron Star

A star is dying. Its core collapses such that its radius goes from 6,000 km to about 40 km. Assume the core does not lose mass.

- If it were rotating at an angular speed of  $0.12 \text{ rad/s}$ , what is its speed now?
- How many times per second does this revolve?



$$I = \frac{2}{5} m R^2$$

### Clicker Question 8:

A star is dying. Its core collapses such that its radius goes from 6,000 km to about 40 km. If it was rotating at an angular speed of 0.12 rad/s, what is its speed now? Assume no outside torques act and the core does not lose mass.

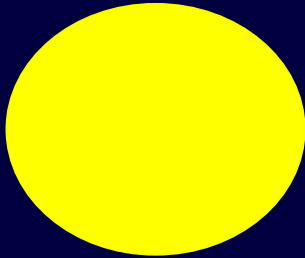
- (a) 2700 rad/s
- (b) 4600 rad/s
- (c) 436 rad/s
- (d) 18 rad/s
- (e) 0.12 rad/s

No external torques, ang mom is conserved

# Rotating Neutron Star

$$\omega_i = 0.12 \frac{\text{rad}}{\text{s}}$$

What is the angular momentum for the star before and after?



$$L_i = \frac{2}{5} m R_i^2 \omega_i$$

$$L_i = L_f$$

$$\frac{2}{5} m R_i^2 \omega_i = \frac{2}{5} m R_f^2 \omega_f$$



$$L_f = \frac{2}{5} m R_f^2 \omega_f$$

$$\omega_f = \left( \frac{R_i}{R_f} \right)^2 \omega_i$$

$$\omega_f = 2700 \frac{\text{rad}}{\text{s}}$$



## Rotating Neutron Star

(b) How many times per second does this revolve?

1.3 h + house

$$\omega = 2700 \frac{\text{rad}}{\text{s}}$$

$$f = \frac{2700 \frac{\text{rad}}{\text{s}}}{2\pi} = 430 \text{ Hz}$$

