Physics 2A: Lecture 17

Today's Agenda

CH 15

- Simple harmonic motion
  - Definition
  - Period and frequency
  - Position, velocity, and acceleration
  - Period of a mass on a spring
  - Vertical spring
- Energy and simple harmonic motion
  - Energy of a spring force

Periodic Motion

- Motion that repeats itself over and over

- Characterized by two pieces of information:

  - Period (T): time it takes to complete one cycle
    - Unit: seconds

  - Frequency (f): number of cycles per unit of time
    - Unit: Hz = seconds^{-1}

$$T = \frac{1}{f}$$

Clicker Question 1:

An object is undergoing periodic motion and takes 10 s to undergo 20 complete oscillations. What is the period and frequency of the object?

(a) T = 10 s, f = 2 Hz

(b) T = 2 s, f = 0.5 Hz

(c) T = 0.5 s, f = 2 Hz

(d) T = 0.5 s, f = 20 Hz

(e) T = 10 s, f = 0.5 Hz

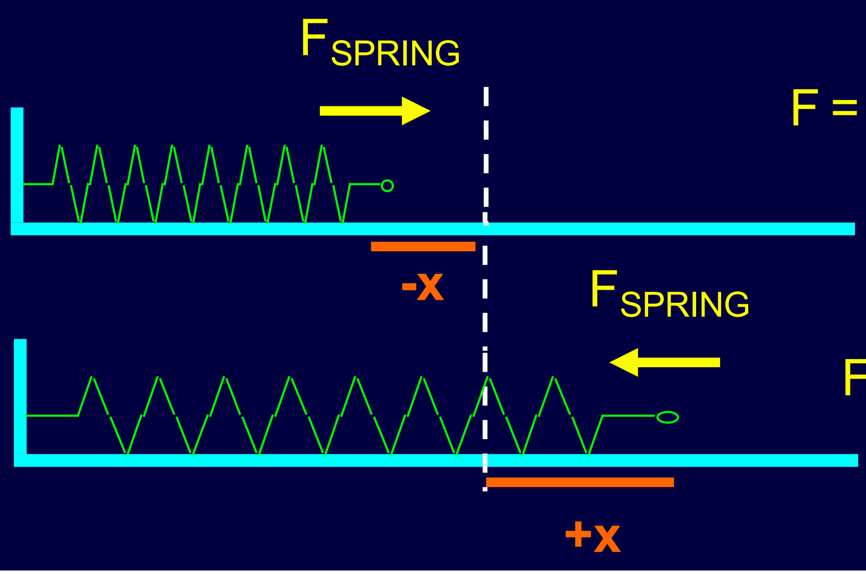
Simple Harmonic Motion

- Particular type of periodic motion
- Very common type of motion
  - Motion due to a spring
  - Motion of a pendulum (small angles)
  - Motion of atoms in molecules

- SHM requires a restoring force

Restoring Force: Hooke's Law

- $F = -kx$
- x is distance spring is displaced from its relaxed length
- k is the spring constant (how stiff the spring is)
- Restoring force is proportional to displacement
- Restoring force is opposite in direction to displacement



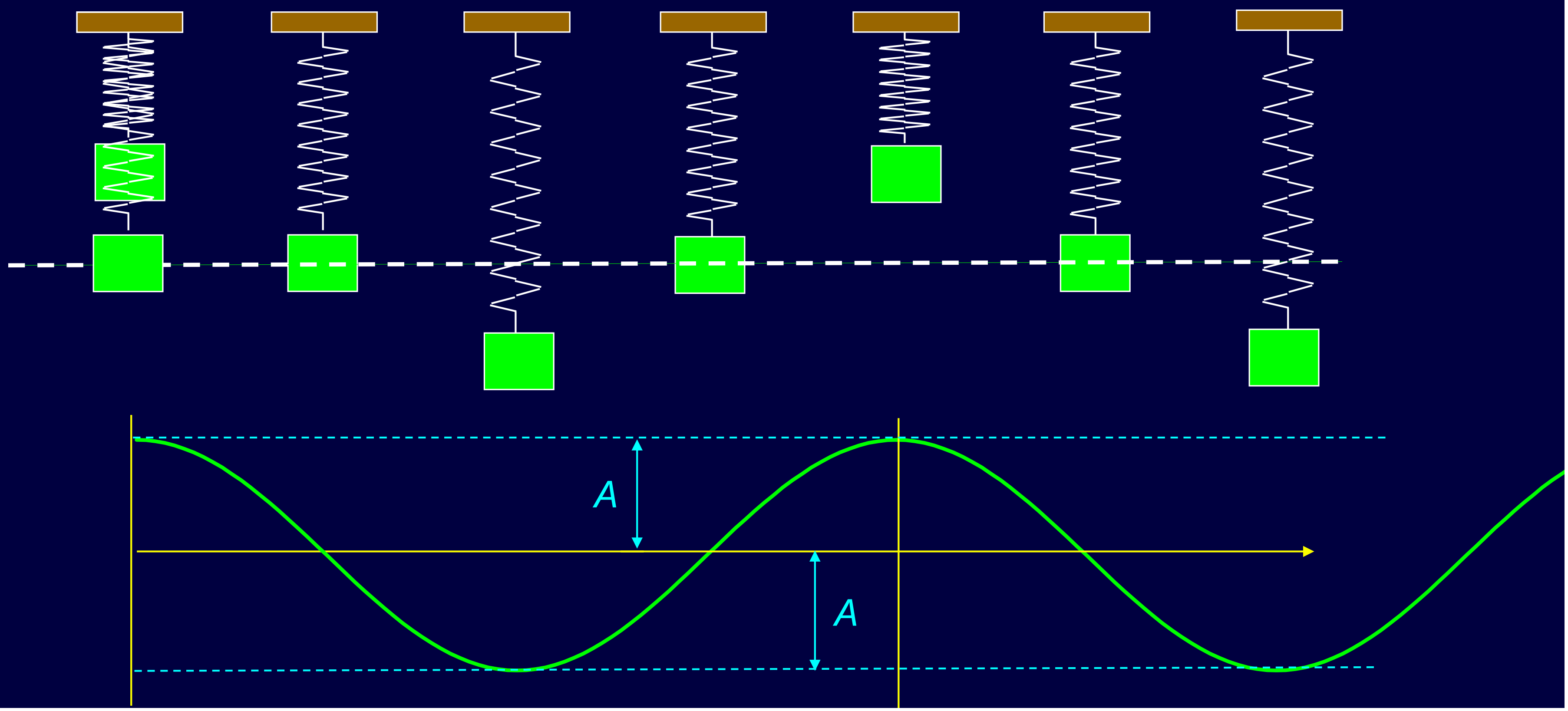
$F = -k(-x) = +kx$

$F = -k(+x) = -kx$

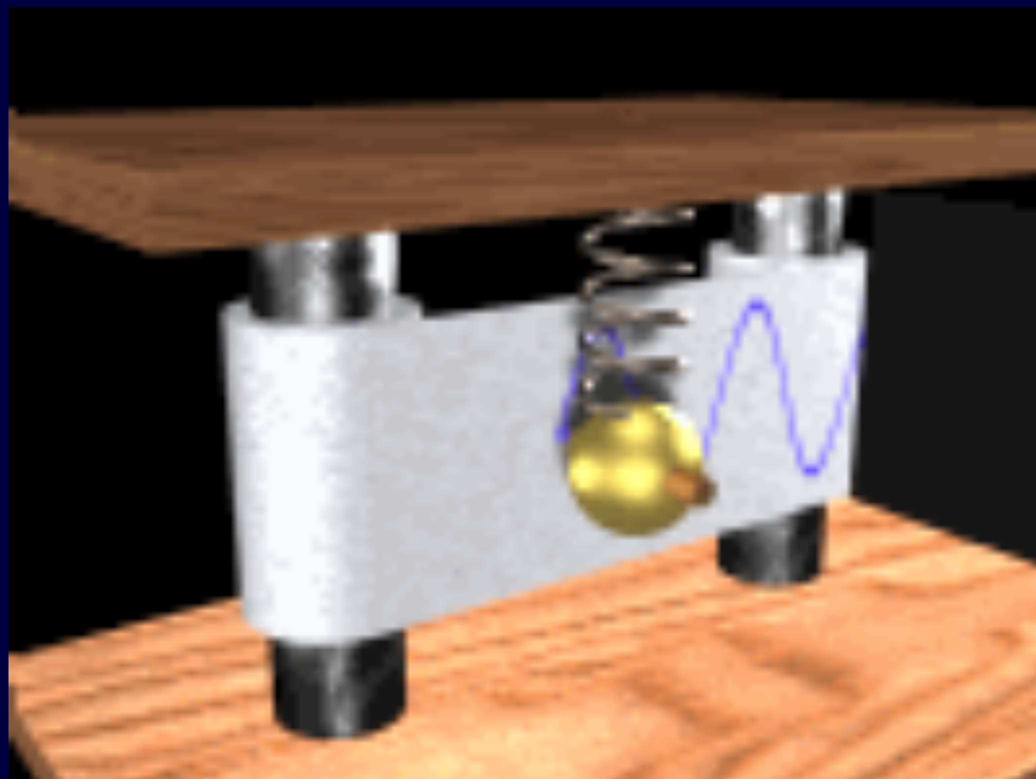
Simple Harmonic Motion

Let's investigate this SHM a bit:

$$y = A\cos\left(\frac{2\pi}{T}t\right)$$



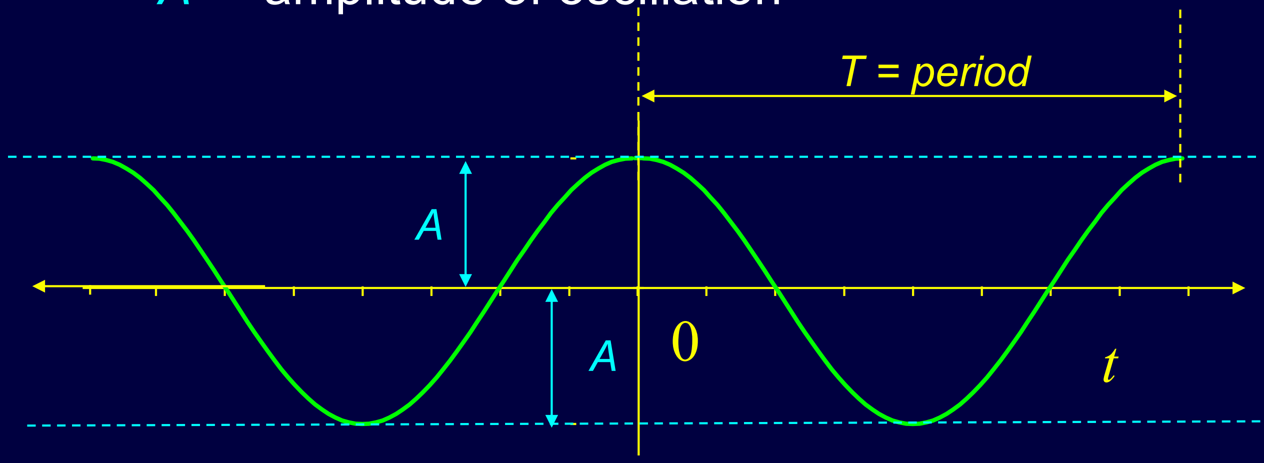
SHM



Position

- Drawing of: $x = A\cos\left(\frac{2\pi}{T}t\right)$

- *A* = amplitude of oscillation



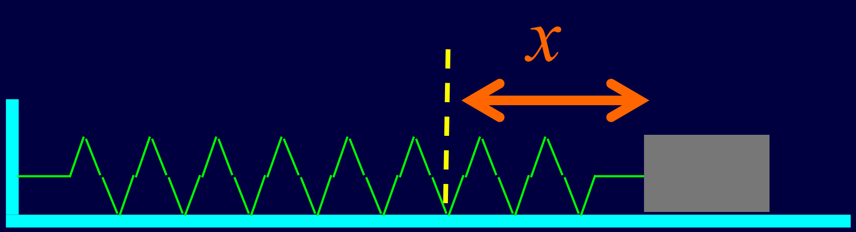
$$x = A\cos\left(\frac{2\pi}{T}0\right) = A$$

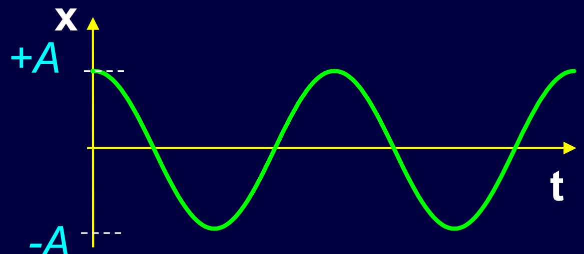
$$x = A\cos\left(\frac{2\pi}{T}(t+T)\right) = A\cos\left(\frac{2\pi}{T}t\right)$$

Clicker Question 2:

A block on a spring oscillates back & forth with simple harmonic motion of amplitude *A*. A plot of displacement (x) versus time (t) is shown below. At what points during its oscillation is the speed of the block biggest?

a) When x = +A or -A (i.e. maximum displacement)
b) When x = 0 (i.e. zero displacement)
c) The speed of the mass is constant





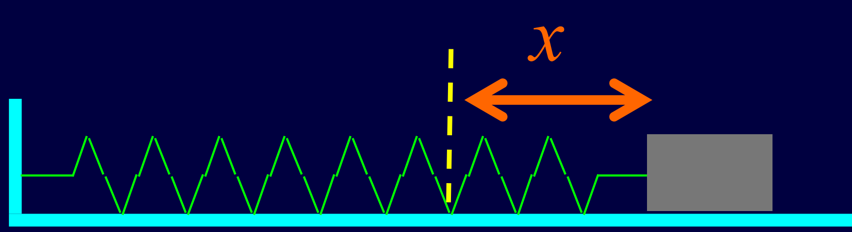
Clicker Question 3:

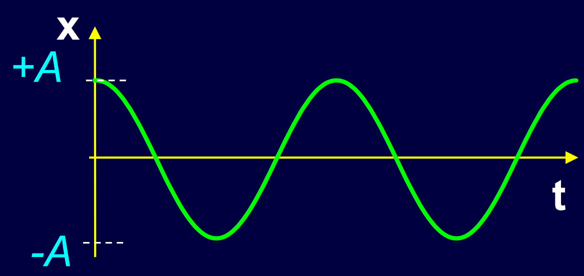
A block on a spring oscillates back & forth with simple harmonic motion of amplitude *A*. A plot of displacement (x) versus time (t) is shown below. At what points during its oscillation is the magnitude of the acceleration of the block biggest?

(a) When x = +A or -A (i.e. maximum displacement)

(b) When x = 0 (i.e. zero displacement)

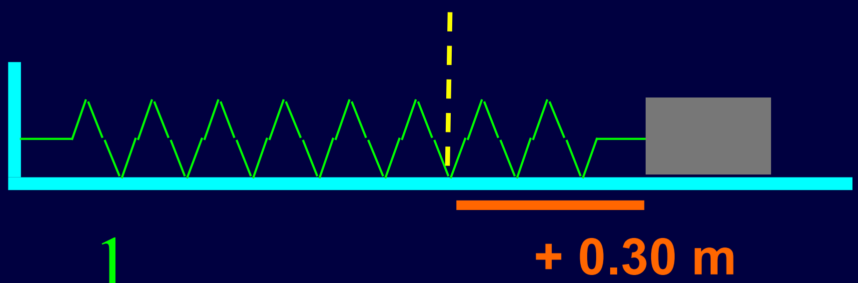
(c) The acceleration of the mass is constant





Example 1

A block is pulled 0.30 m from its equilibrium position and let go.
(a) Find the block's equation of motion.
(b) What is the block's position after 0.5 s?
The frequency of oscillation for the block/spring system is 0.77 Hz.



$$T = \frac{1}{0.77\,\mathrm{Hz}} = 1.3\,\mathrm{s}$$

(a) $$x = 0.30\,\mathrm{m}\cos\left(\frac{2\pi}{1.3\,\mathrm{s}}t\right)$$

Remember to use radians!

(b) $$x = 0.30\,\mathrm{m}\cos\left(\frac{2\pi}{1.3\,\mathrm{s}}0.5\,\mathrm{s}\right) = -0.22\,\mathrm{m}$$

Angular Frequency vs. Frequency

To keep things simple we'll express our function in terms of the angular frequency

$$\omega = 2\pi f \qquad \omega = \frac{2\pi}{T}$$

$$x = A\cos\left(\frac{2\pi}{T}t\right) \qquad x = A\cos(\omega t)$$

Clicker Question 5:

What is the time derivative of the function $f(t) = A \cos(\omega t)$?

(a) $-A \cos(\omega t)$

(b) $-\omega A \cos(\omega t)$

(c) $A \sin(\omega t)$

(d) $\omega A \sin(\omega t)$

(e) $-\omega A \sin(\omega t)$

Simple Harmonic Motion:

$x(t) = [A]\cos(\omega t)$

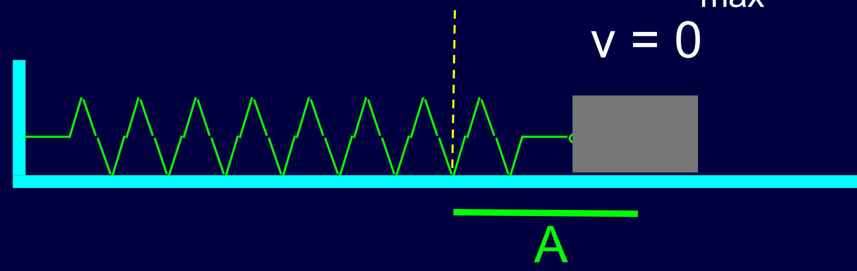
$v(t) = -[A\omega]\sin(\omega t)$

$a(t) = -[A\omega^2]\cos(\omega t)$

$t = 0$

$x = x_{max} = A$

$v = 0$



$x(t) = [A]\sin(\omega t)$

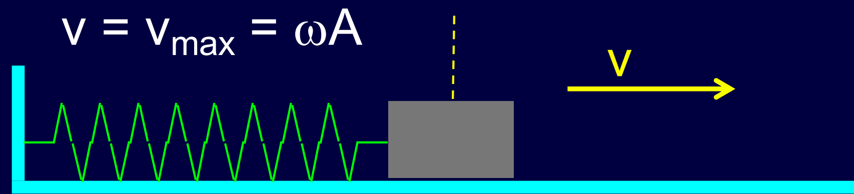
$v(t) = [A\omega]\cos(\omega t)$

$a(t) = -[A\omega^2]\sin(\omega t)$

$t = 0$

$x = 0$

$v = v_{max} = \omega A$



Example 2

An object experiences SHM with an equation of position $x = (2\text{ m}) \cos(30\, t)$.

What is the period of oscillation?

$$T = 2\pi/\omega = 0.209\text{ s}$$

What is the maximum velocity?

$$V_{max} = \omega A = 60\text{ m/s}$$

What is the maximum displacement of the object?

± 2 m (it's just the amplitude)

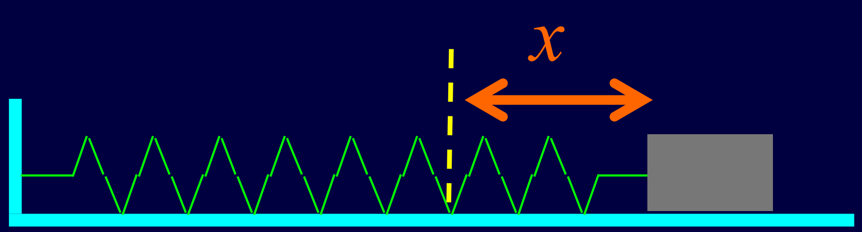
What is the maximum acceleration?

$$a_{max} = \omega^2 A = 30^2(2) = 1800\text{ m/s}^2$$

Clicker Question 4:

Object A is attached to spring A and is moving in simple harmonic motion. Object B is attached to spring B and is moving in simple harmonic motion. The period and the amplitude of object B are both two times the corresponding values for object A. How do the maximum speeds of the two objects compare?

a) The maximum speed of A is one fourth that of object B.
b) The maximum speed of A is one half that of object B.
c) The maximum speed of A is the same as that of object B.
d) The maximum speed of A is two times that of object B.
e) The maximum speed of A is four times that of object B.



Clicker Question 5:

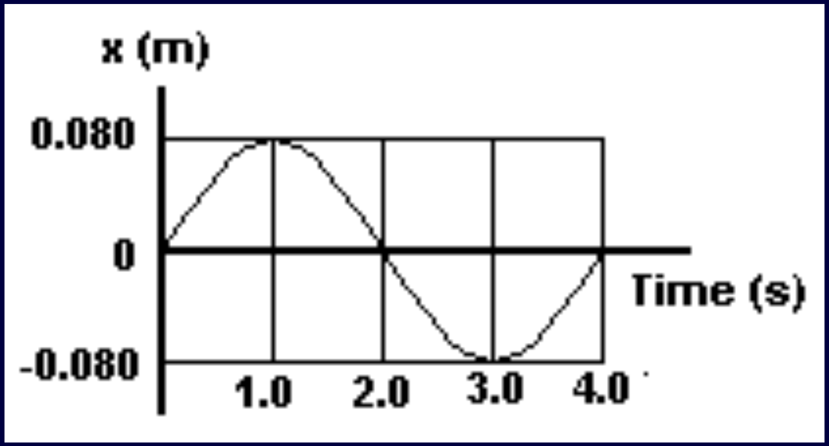
An 0.80 kg object is attached to one end of a spring, and the system is set into simple harmonic motion. The displacement of *x* of the object as a function of time is shown in the drawing. What is the correct equation for *x*?

(a) $x = 0.080\cos(\pi t)$

(b) $x = 0.160\sin(\pi t)$

(c) $x = 0.080\sin\left(\dfrac{\pi}{2}t\right)$

(d) $x = 0.160\sin\left(\dfrac{\pi}{2}t\right)$



Simple Harmonic Motion:

$x(t) = [A]\cos(\omega t)$
$v(t) = -[\omega A]\sin(\omega t)$
$a(t) = -[\omega^2 A]\cos(\omega t)$

OR

$x(t) = [A]\sin(\omega t)$
$v(t) = [\omega A]\cos(\omega t)$
$a(t) = -[\omega^2 A]\sin(\omega t)$

$x_{max} = A$
$v_{max} = A\omega$
$a_{max} = A\omega^2$

Period = T (seconds per cycle)
Frequency = f = 1/T (cycles per second)
Angular frequency = $\omega = 2\pi f = 2\pi/T$

$$x(t) = A\cos(\omega t + \phi)$$

$$v(t) = -\omega A\sin(\omega t + \phi)$$

$$a(t) = -\omega^2 A\cos(\omega t + \phi)$$

Simple Harmonic Motion:

$t = 0$

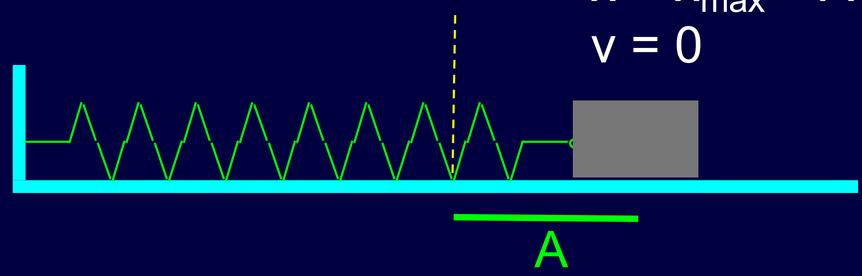
$x = x_{max} = A$

$v = 0$

$x(t) = [A]\cos(\omega t)$

$v(t) = -[A\omega]\sin(\omega t)$

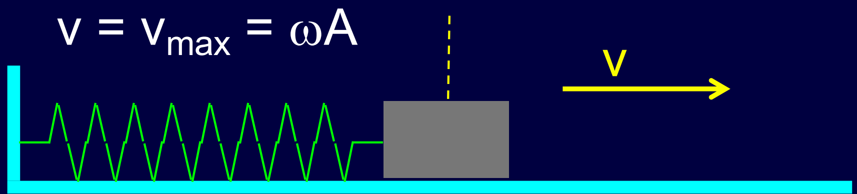
$a(t) = -[A\omega^2]\cos(\omega t)$



$t = 0$

$x = 0$

$v = v_{max} = \omega A$



$x(t) = [A]\sin(\omega t)$

$v(t) = [A\omega]\cos(\omega t)$

$a(t) = -[A\omega^2]\sin(\omega t)$

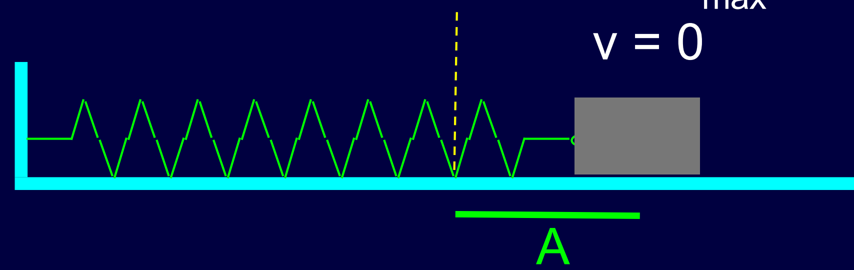
Simple Harmonic Motion:

$$x(t) = A\cos(\omega t + \phi) \Rightarrow x(0) = A\cos(\phi)$$

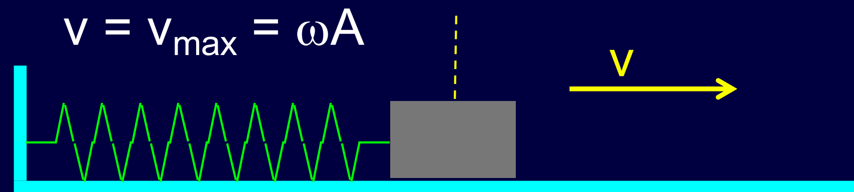
$$\phi = 0$$

$$x(0) = A\cos(0) = A$$

t = 0
x = x_{max} = A
v = 0



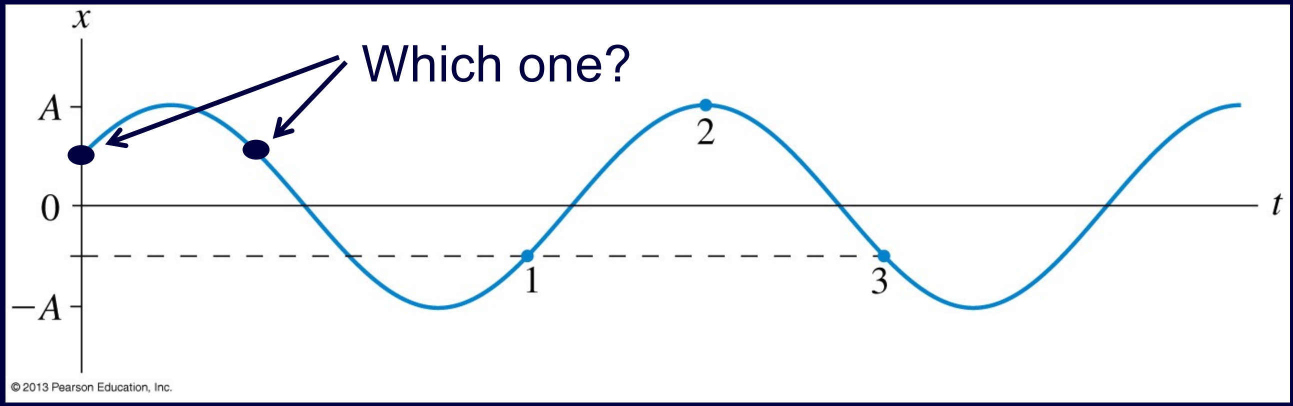
t = 0
x = 0
v = v_{max} = ωA



$$\phi = -\frac{\pi}{2}$$

$$x(0) = A\cos\left(-\frac{\pi}{2}\right) = 0$$

Phase Constant



$$x(t) = A\cos(\omega t + \phi)$$

$$x(0) = A\cos(\phi) = \frac{A}{2}$$

$$\cos(\phi) = \frac{1}{2}$$

$$\phi = \pm 1.047 = \pm\frac{\pi}{3}$$

$$v(t) = -\omega A\sin(\omega t + \phi)$$

$$v(0) = -\omega A\sin(\phi)$$

$$v(0) = -\omega A\sin\left(-\frac{\pi}{3}\right) = \text{positive}$$

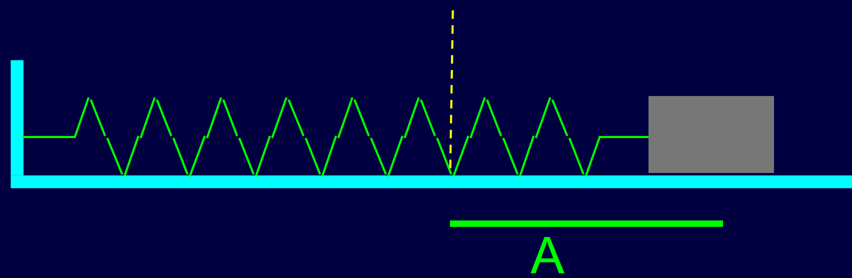
$$x(t) = A\cos\left(\omega t - \frac{\pi}{3}\right)$$

Period for a Mass on a Spring

- What can Newton's second law tell us about SHM?

$$\sum \mathrm{F} = \mathrm{m}a$$

$$-kx = \mathrm{m}a$$



Period for a Mass on a Spring

- What can Newton's second law tell us about SHM?

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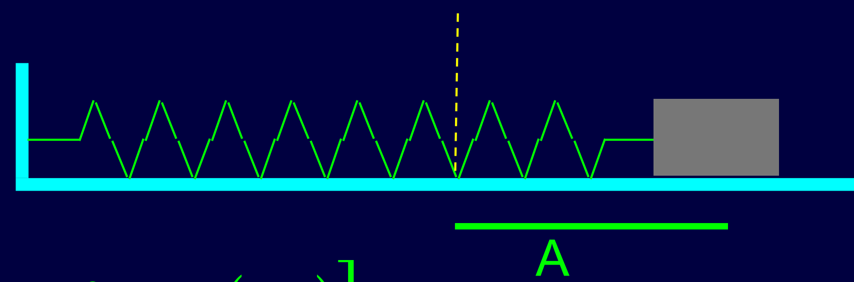


$$-k\left[A\cos(\omega t)\right] = \mathrm{m}\left[-A\omega^2\cos(\omega t)\right]$$

$$k = \mathrm{m}\omega^2$$

$$\boxed{\omega = \sqrt{\frac{\mathrm{k}}{\mathrm{m}}}} \qquad \therefore \boxed{\mathrm{T} = 2\pi\sqrt{\frac{m}{k}}}$$

Clicker Question 6:

A mass is attached to a spring. I pull it distance of A and it oscillates with frequency f. If I pull it a distance of 2A what will the frequency be?

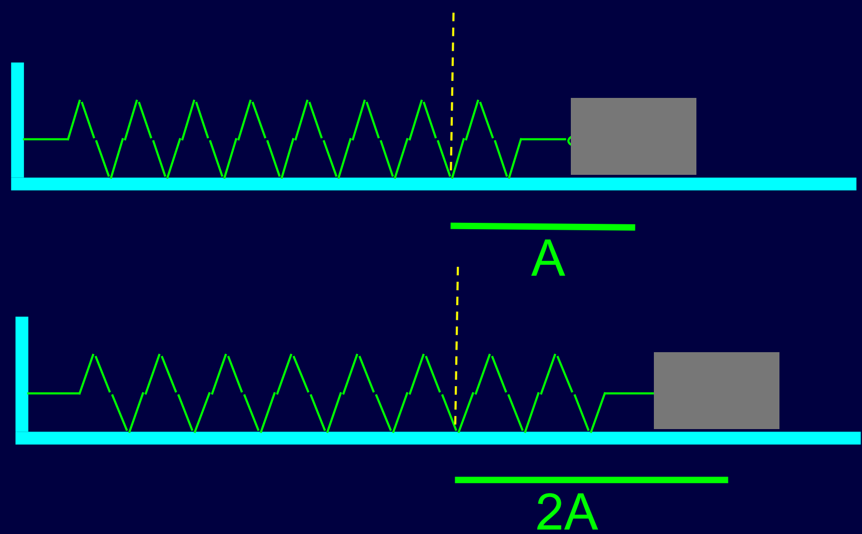
(a) 4f

(b) 2f

(c) f

(d) f/2

(e) f/4



$$\omega = \sqrt{\frac{k}{m}}$$

Demo

Clicker Question 7:

A block of mass m oscillates on a horizontal spring with period $T = 2.0\ \text{s}$. If a second identical block is glued to the top of the first block, the new period will be

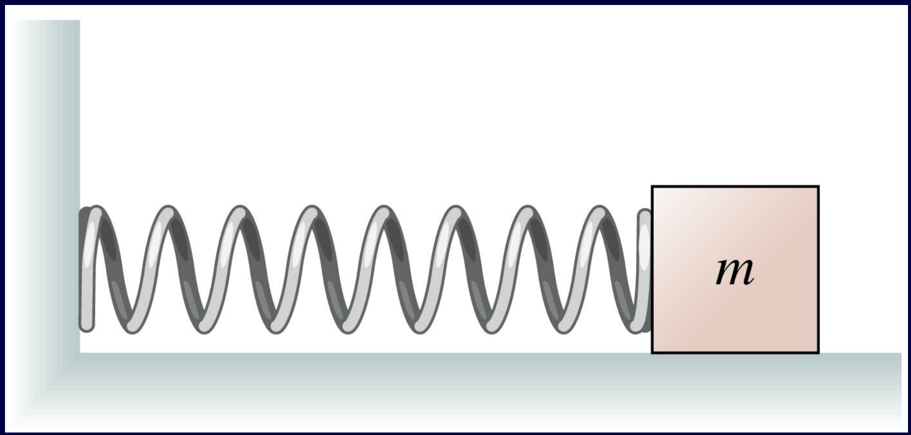
A. $1.0\ \text{s}$.

B. $1.4\ \text{s}$.

C. $2.0\ \text{s}$.

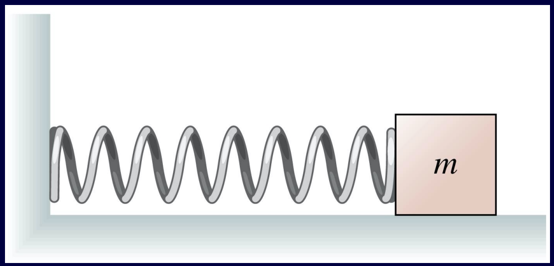
D. $2.8\ \text{s}$.

E. $4.0\ \text{s}$.



Clicker Question 8:

A block of mass m oscillates on a horizontal spring with period $T = 2.0$ s. If a second identical block is glued to the top of the first block, the new period will be



$$T_i = 2\pi\sqrt{\frac{m_i}{k}}$$

$$T_f = 2\pi\sqrt{\frac{m_f}{k}}$$

$$\frac{T_f}{T_i} = \sqrt{\frac{m_f}{m_i}}$$

$$\frac{T_f}{T_i} = \sqrt{\frac{2m}{m}}$$

$$T_f = \sqrt{2}T_i$$

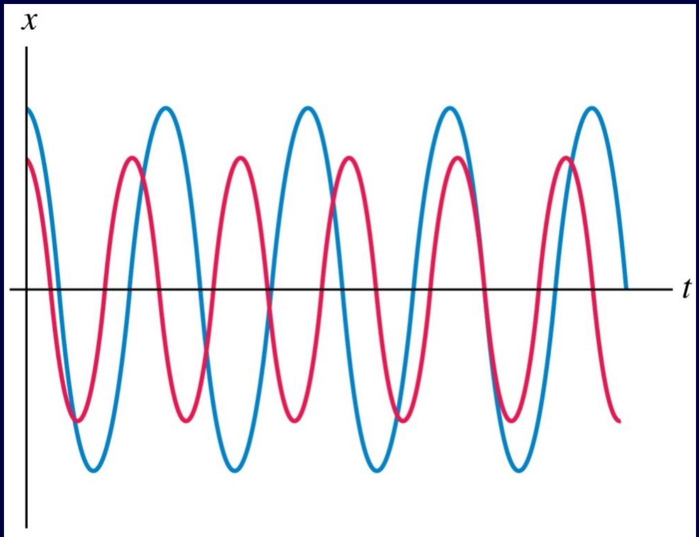
Clicker Question 9:

Two identical blocks oscillate on different horizontal springs. Which spring has the larger spring constant?

A. The red spring.

B. The blue spring.

C. There's not enough information to tell.



$$T_R < T_B$$

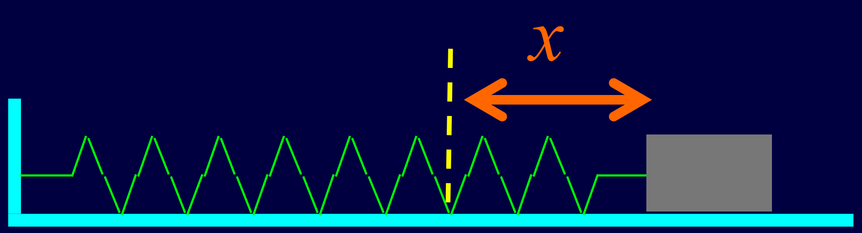
$$\omega_R > \omega_B$$

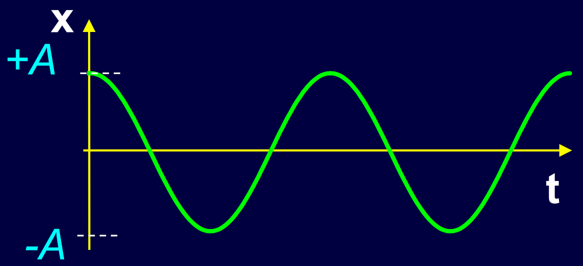
$$\omega = \sqrt{\frac{k}{m}}$$

Clicker Question 10:

A mass on a spring oscillates back & forth with simple harmonic motion of amplitude *A*. A plot of displacement (x) versus time (t) is shown below. At what points during its oscillation is the total mechanical energy (K+U) of the mass and spring a maximum? (Ignore friction).

a) When x = +A or -A (i.e. maximum displacement)
b) When x = 0 (i.e. zero displacement)
c) The mechanical energy of the system is constant





Energy Conservation

- If there are no non-conservative forces acting, the mechanical energy will be conserved:

$$E = K + U = \tfrac{1}{2} m v^2 + \tfrac{1}{2} k x^2$$

- At maximum displacement, x=*A*, v = 0:

$$E = \tfrac{1}{2} k A^2$$

$$v_{\mathrm{max}} = \omega A$$

- At zero displacement, x = 0:

$$E = \tfrac{1}{2} m {v_{\mathrm{max}}}^2 = \tfrac{1}{2} m \omega^2 A^2$$

$$E = \tfrac{1}{2} m \frac{k}{m} A^2 = \tfrac{1}{2} k A^2$$

$$\omega^2 = \frac{\mathrm{k}}{\mathrm{m}}$$

Clicker Question 11:

A mass oscillates in simple harmonic motion with amplitude A. If the mass is doubled, but the amplitude is not changed, what will happen to the total mechanical energy of the system?

a) total energy will increase

b) total energy will not change

c) total energy will decrease

$$E = \tfrac{1}{2} kA^2$$

$$\omega = \sqrt{\frac{k}{m}}$$

Pre-class Quiz:

A block oscillating on a spring has an amplitude of 13cm. What will the block's amplitude be if its total energy is doubled?

$$E = \tfrac{1}{2} k A_i^{\,2}$$

$$2E = \tfrac{1}{2} k A_f^{\,2}$$

$$\frac{E}{2E} = \frac{\tfrac{1}{2} k A_i^{\,2}}{\tfrac{1}{2} k A_f^{\,2}}$$

$$A_f^2 = 2A_i^2$$

$$A_f = \sqrt{2} A_i$$

Example

A block of mass 5.0 kg on a frictionless surface is attached to a horizontal spring with k = 32 N/m. If the block is pulled 0.20 m from the equilibrium position of the spring, what velocity will it have when it is 0.10 m from the equilibrium position?

$$K_i + U_i = K_f + U_f$$

$$\cancel{\tfrac{1}{2} m v_i^2} + \tfrac{1}{2} k x_i^2 = \tfrac{1}{2} m v_f^2 + \tfrac{1}{2} k x_f^2$$

$$v_f = \sqrt{\frac{k}{m}\left(x_i^2 - x_f^2\right)} \qquad = 0.44\,\mathrm{m/s}$$

$$K_i + U_i = \tfrac{1}{2} 32\,\mathrm{N/m}\left(0.20\,\mathrm{m}\right)^2 = 0.64\,\mathrm{J}$$

$$K_f + U_f = \tfrac{1}{2} 32\,\mathrm{N/m}\left(0.10\,\mathrm{m}\right)^2 + \tfrac{1}{2} 5\,\mathrm{kg}\left(0.44\,\mathrm{m/s}\right)^2 = 0.64\,\mathrm{J}$$

Pendulum Motion

- Free-body diagram for pendulum:

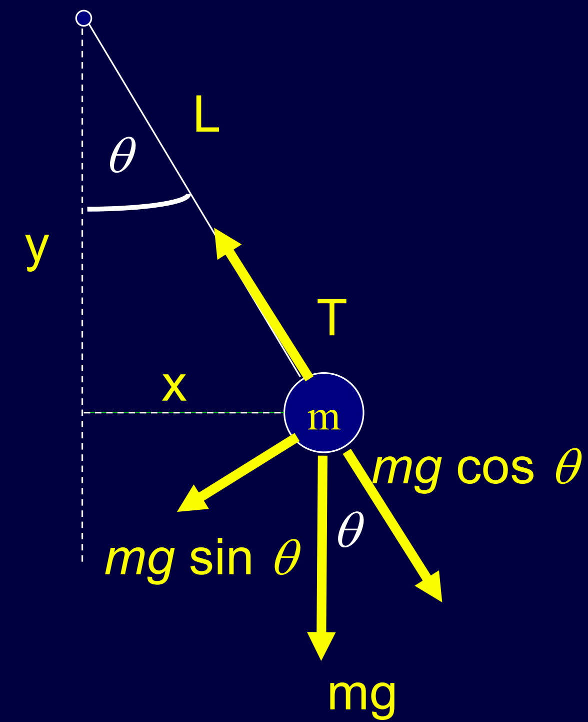
$$F_{\tan} = mg\sin\theta$$

For small oscillations

$$F_{\tan} \approx mg\theta$$

$$F_{\tan} = mg\frac{s}{L} = \frac{mg}{L}s$$

$$F = kx \qquad k = \frac{mg}{L}$$



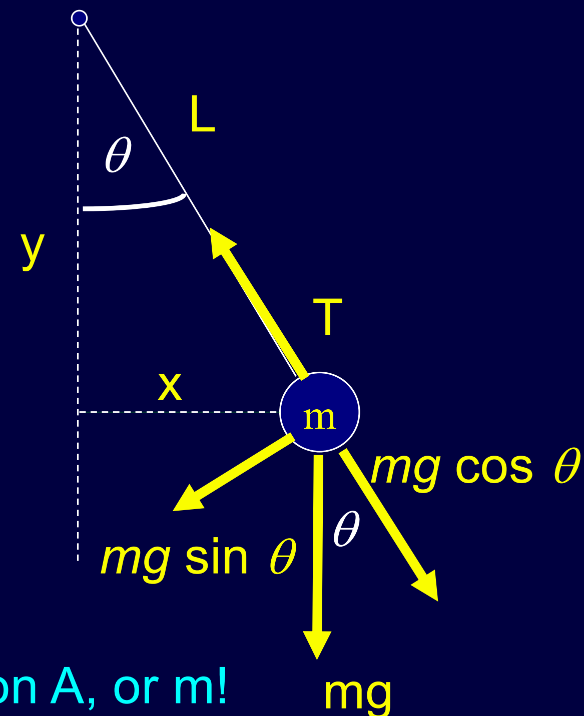
Pendulum Motion

- Free-body diagram for pendulum:

$$k = \frac{mg}{L}$$

$$\omega = \sqrt{\frac{\mathrm{k}}{\mathrm{m}}} = \sqrt{\frac{\mathrm{g}}{\mathrm{L}}}$$

$$\mathrm{T} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{L}{g}}$$



Period and frequency do not depend on A, or m!
(For small oscillations)

Example:

A pendulum on Planet X, where the value of *m* is unknown, oscillates with a period T_1 = 2.20s . Note that you do not know the value of *m*, *L*, or *g*, so do not assume any specific values. The required analysis involves thinking about ratios.

What is the period if Its length is doubled?

$$\mathrm{T}_1 = 2\pi\sqrt{\frac{L_1}{g}}$$

$$\frac{\mathrm{T}_1}{\mathrm{T}_2} = \sqrt{\frac{L_1}{L_2}}$$

$$\frac{\mathrm{T}_1}{\mathrm{T}_2} = \sqrt{\frac{L}{2L}}$$

$$\mathrm{T}_2 = 2\pi\sqrt{\frac{L_2}{g}}$$

Period and frequency do not depend on A, or m!
(For small oscillations)