Physics 2A: Lecture 17

Today's Agenda

CH 15

- Simple harmonic motion
  - Definition
  - Period and frequency
  - Position, velocity, and acceleration
  - Period of a mass on a spring
  - Vertical spring
- Energy and simple harmonic motion
  - Energy of a spring force

Periodic Motion

- Motion that repeats itself over and over
- Characterized by two pieces of information:
  - Period (T): time it takes to complete one cycle
    - Unit: seconds
  - Frequency (f): number of cycles per unit of time
    - Unit: $\boxed{\text{Hz}}$ = seconds^{-1}

$$T = \frac{1}{f}$$

$$f = \frac{1}{T} = \frac{1}{s} = s^{-1}$$

Clicker Question 1:

An object is undergoing periodic motion and takes 10 s to undergo 20 complete oscillations. What is the period and frequency of the object?

(a) T = 10 s, f = 2 Hz

(b) T = 2 s, f = 0.5 Hz

(c) T = 0.5 s, f = 2 Hz

(d) T = 0.5 s, f = 20 Hz

(e) T = 10 s, f = 0.5 Hz

$$f = \frac{20 \text{ osullations}}{10 s} = 2 \text{ Hz}$$

$$T = \frac{1}{f} = \frac{1}{2} s$$

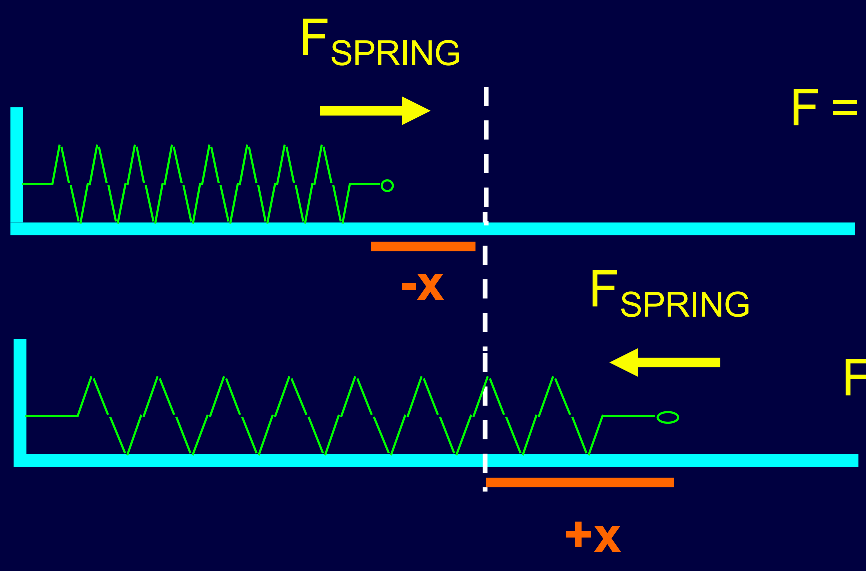
Simple Harmonic Motion

- Particular type of periodic motion
- Very common type of motion
  - Motion due to a spring
  - Motion of a pendulum (small angles)
  - Motion of atoms in molecules

- SHM requires a restoring force

Restoring Force: Hooke's Law

- $F = -kx$
- x is distance spring is displaced from its relaxed length
- k is the spring constant (how stiff the spring is)
- Restoring force is proportional to displacement
- Restoring force is opposite in direction to displacement



$F = -k(-x) = +kx$

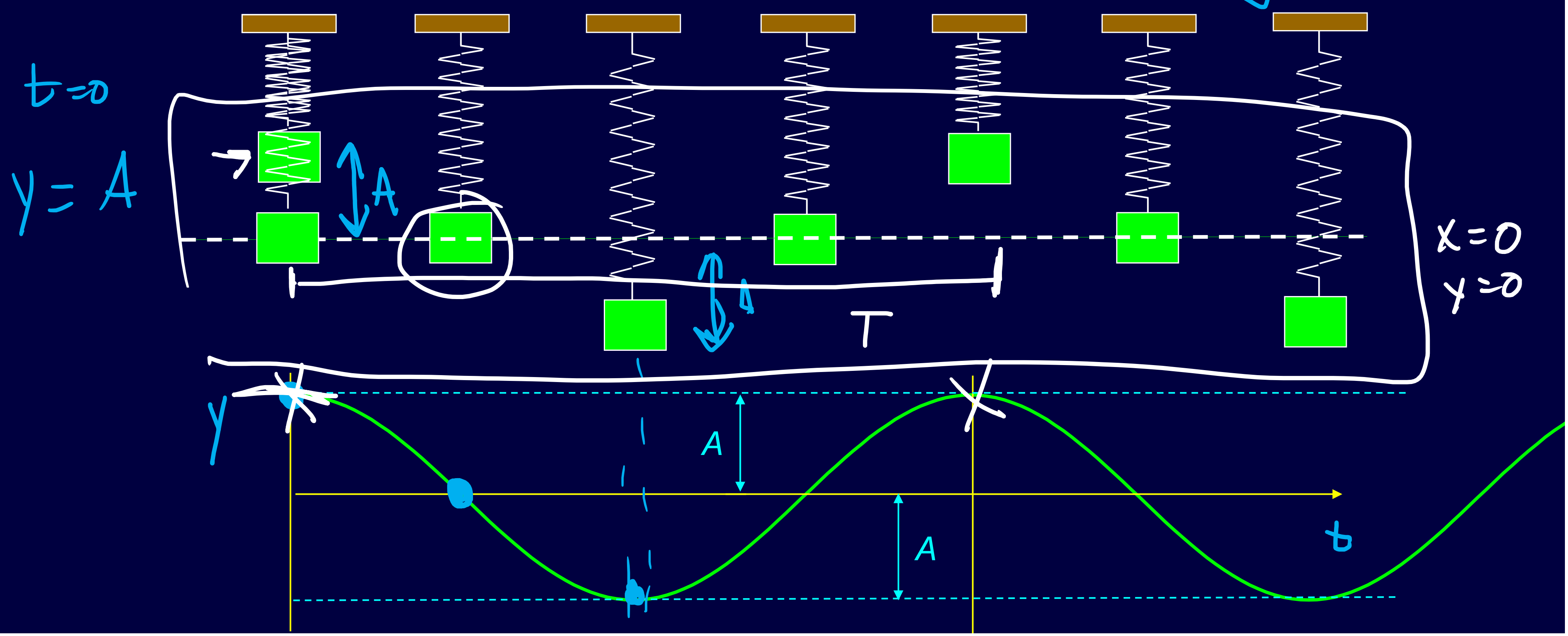
$F = -k(+x) = -kx$

model

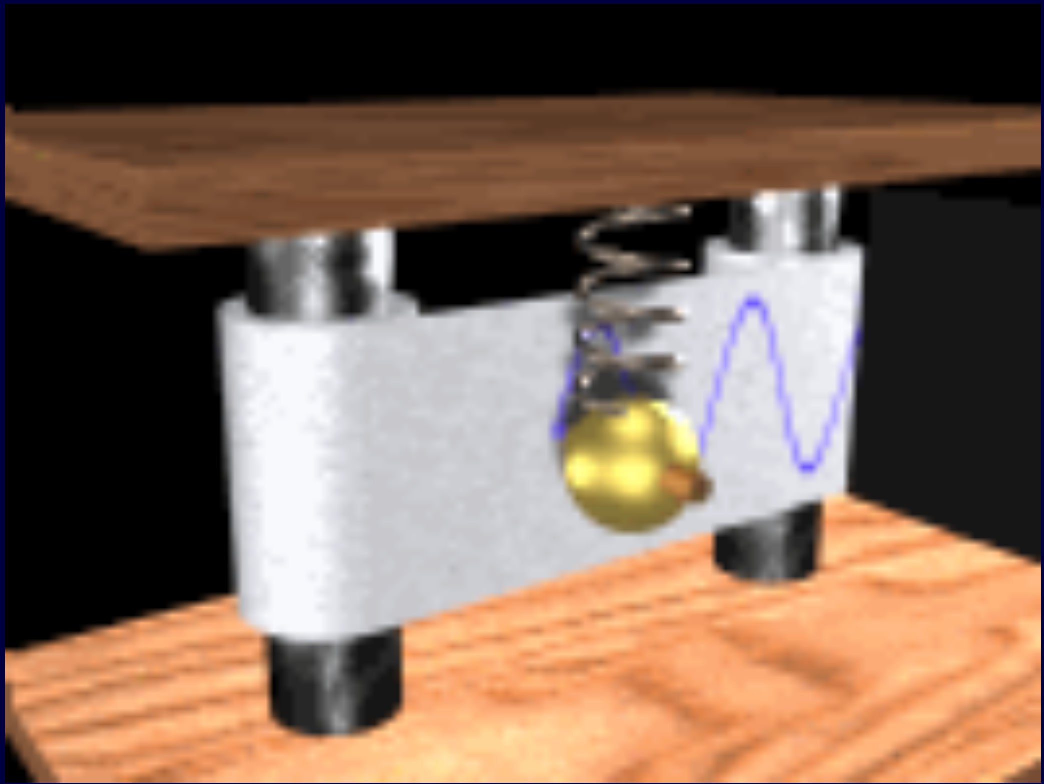
Simple Harmonic Motion

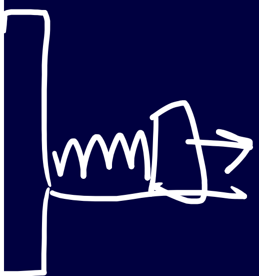
Let's investigate this SHM a bit:

$$y = A\cos\left(\boxed{\frac{2\pi}{T}}t\right)$$



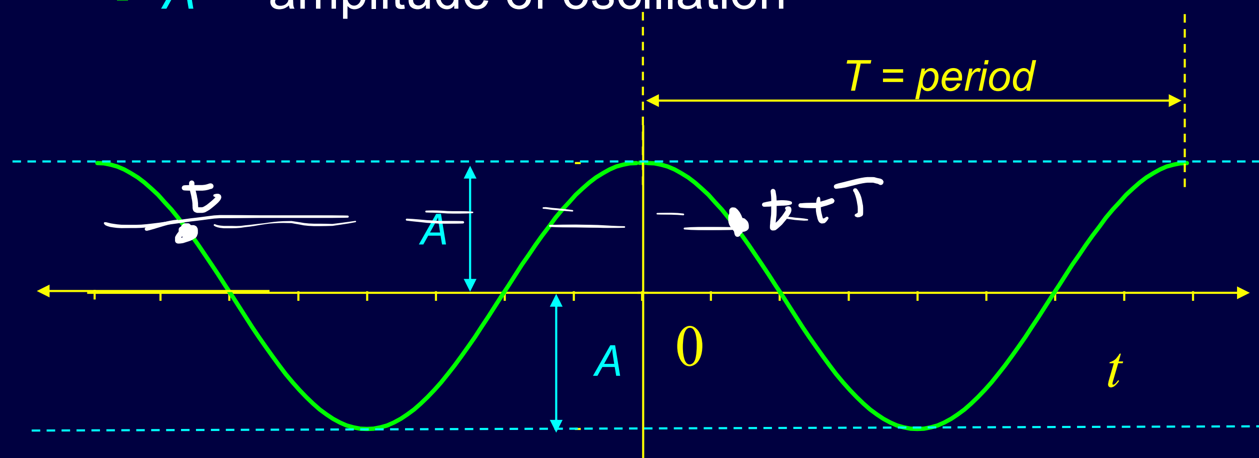
SHM





Position

- Drawing of: $x = A\cos\left(\frac{2\pi}{T}t\right)$
- A = amplitude of oscillation



$$x = A\cos\left(\frac{2\pi}{T}0\right) = A$$

$$x = A\cos\left(\frac{2\pi}{T}(t+T)\right) = A\cos\left(\frac{2\pi}{T}t\right)$$

$$= A\cos\left(\frac{2\pi t}{T} + 2\pi\right) =$$

Clicker Question 2:

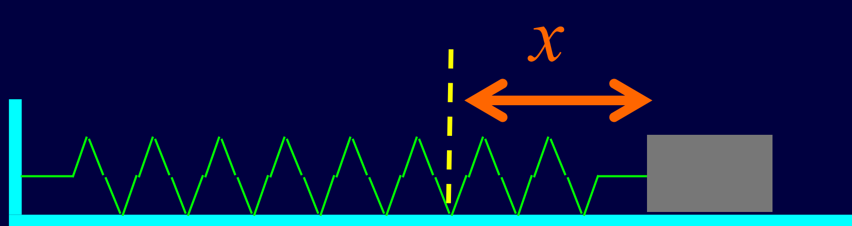
A block on a spring oscillates back & forth with simple harmonic motion of amplitude *A*. A plot of displacement (x) versus time (t) is shown below. At what points during its oscillation is the speed of the block biggest?

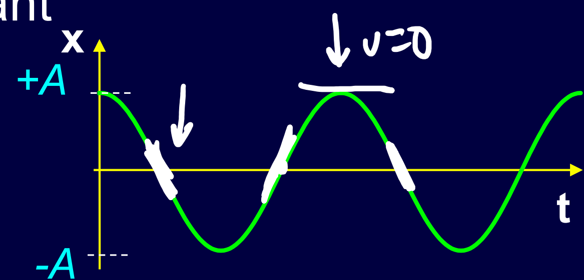
$$v = \frac{dx}{dt}$$

a) When x = +A or -A (i.e. maximum displacement)

b) When x = 0 (i.e. zero displacement)

c) The speed of the mass is constant





Clicker Question 3:

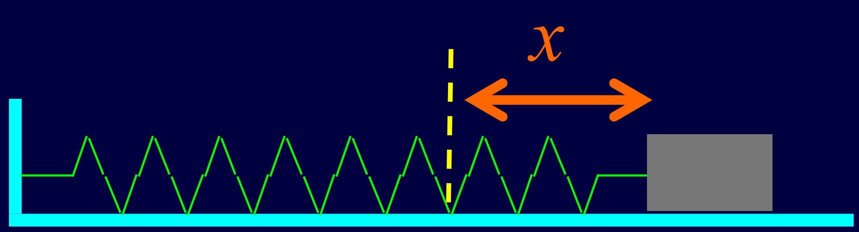
A block on a spring oscillates back & forth with simple harmonic motion of amplitude *A*. A plot of displacement (x) versus time (t) is shown below. At what points during its oscillation is the magnitude of the acceleration of the block biggest?

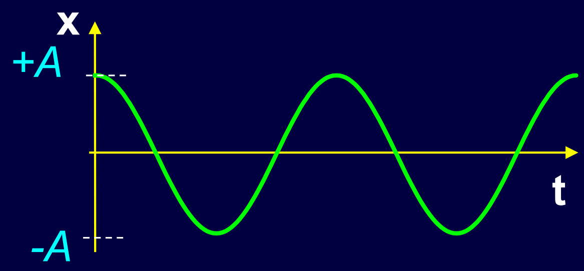
(a) When x = +A or -A (i.e. maximum displacement)

(b) When x = 0 (i.e. zero displacement)

(c) The acceleration of the mass is constant

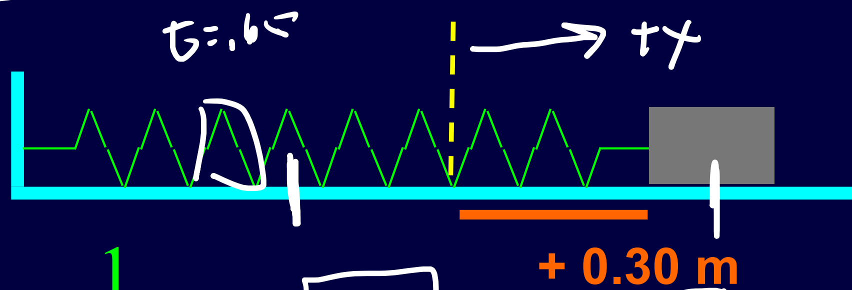
$E_{mech} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$





Example 1

A block is pulled 0.30 m from its equilibrium position and let go.
(a) Find the block's equation of motion.
(b) What is the block's position after 0.5 s?
The frequency of oscillation for the block/spring system is 0.77 Hz.



$$T = \frac{1}{0.77\,\text{Hz}} = \boxed{1.3\,\text{s}}$$

(a) $x = 0.30\,\text{m}\cos\left(\frac{2\pi}{1.3\,\text{s}}t\right)$

(b) $x = 0.30\,\text{m}\cos\left(\frac{2\pi}{1.3\,\text{s}}0.5\,\text{s}\right) = -0.22\,\text{m}$

Remember to use radians!

Angular Frequency vs. Frequency

ω

f

T

To keep things simple we'll express our function in terms of the angular frequency

$$\omega = 2\pi f \qquad \omega = \frac{2\pi}{T}$$

$$x = A\cos\left(\frac{2\pi}{T}t\right) \qquad x = A\cos(\omega t)$$

Clicker Question 5:

What is the time derivative of the function f(t) = A Cos (ωt)?

(a) -A Cos (ωt)

(b) - ω A Cos (ωt)

(c) A Sin (ωt)

(d) ω A Sin (ωt)

(e) - ω A Sin (ωt) ✓

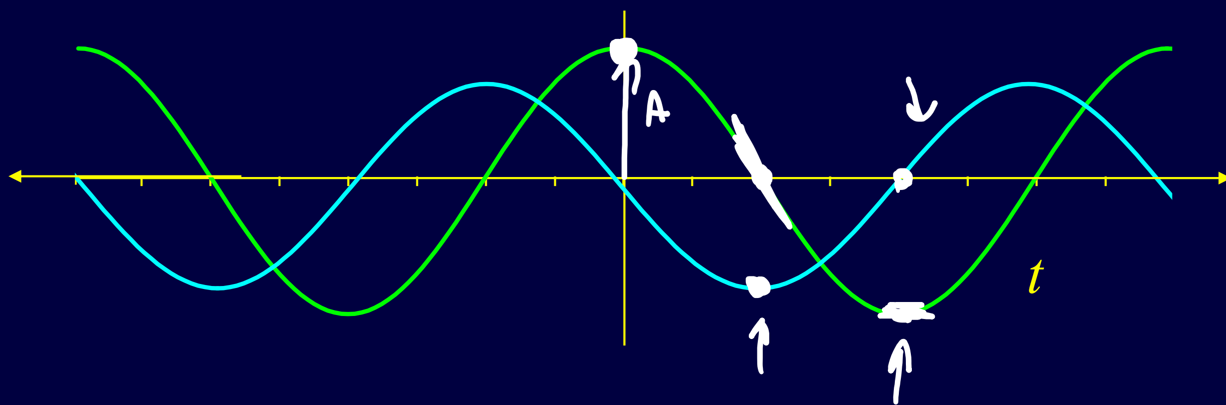
$$\rightarrow x(t) = f(t) = A\cos(\omega t)$$

$$\frac{df}{dt} = -\omega A \sin(\omega t)$$

$$\rightarrow v(t) = -\omega A \sin(\omega t)$$

$$\rightarrow a(t) = -\omega^2 A \cos(\omega t)$$

Position and Velocity



$$x = A\underline{\cos}\,\theta$$

$$v = -\omega A \sin\theta$$

- x is zero when v = +ωA or - ω A
- v is zero when x = +A or -A

Acceleration

x(t) = [A]cos(ωt)

v(t) = -[Aω]sin(ωt)

$x_{max} = \pm A$

$v_{max} = \pm \omega A$

$a_{max} = \pm \omega^2 A$

$$a = -\omega^2 A\cos(\omega t)$$

$$|a_{\max}| = \omega^2 A$$

Simple Harmonic Motion:

Start from $+x_{max}$

$x(t) = [A]\cos(\omega t)$

$v(t) = -[A\omega]\sin(\omega t)$

$a(t) = -[A\omega^2]\cos(\omega t)$

OR

$x(t) = [A]\sin(\omega t)$

$v(t) = [A\omega]\cos(\omega t)$

$a(t) = -[A\omega^2]\sin(\omega t)$

$x_{max} = A$

$v_{max} = A\omega$

$a_{max} = A\omega^2$

Period = T (seconds per cycle)

Frequency = f = 1/T (cycles per second)

Angular frequency = $\omega = 2\pi f = 2\pi/T$

Simple Harmonic Motion:

$x(t) = [A]\cos(\omega t)$

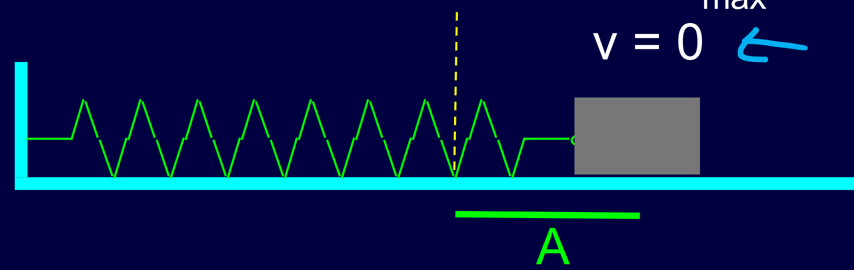
$v(t) = -[A\omega]\sin(\omega t)$

$a(t) = -[A\omega^2]\cos(\omega t)$

$t = 0$

$x = x_{max} = A$

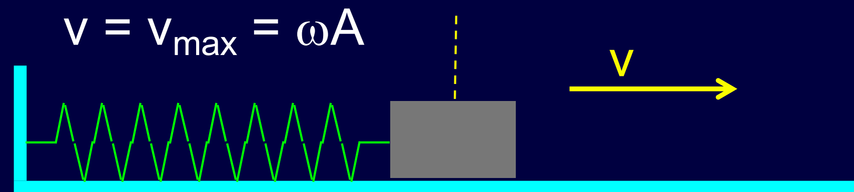
$v = 0$



$t = 0$

$x = 0$

$v = v_{max} = \omega A$



$x(t) = [A]\sin(\omega t)$

$v(t) = [A\omega]\cos(\omega t)$

$a(t) = -[A\omega^2]\sin(\omega t)$

Example 2

$w = \frac{2\pi}{T}$

$T = \frac{2\pi}{w}$

An object experiences SHM with an equation of position $x = (2\text{ m})\cos(30\,t)$.

What is the period of oscillation?

$T = 2\pi/\omega = 0.209$ s

What is the maximum velocity?

$V_{max} = \omega A = 60$ m/s

What is the maximum displacement of the object?

± 2 m (it's just the amplitude)

What is the maximum acceleration?

$a_{max} = \omega^2 A = 30^2(2) = 1800\ \text{m/s}^2$

Clicker Question 4:

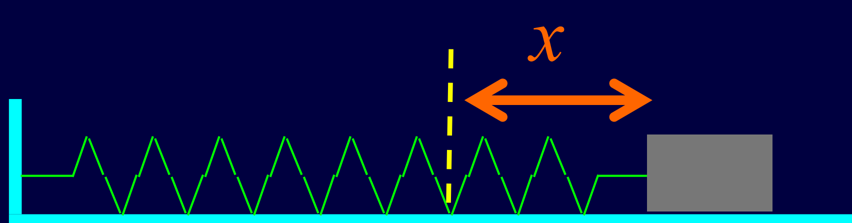
Object A is attached to spring A and is moving in simple harmonic motion. Object B is attached to spring B and is moving in simple harmonic motion. The period and the amplitude of object B are both two times the corresponding values for object A. How do the maximum speeds of the two objects compare?

a) The maximum speed of A is one fourth that of object B.
b) The maximum speed of A is one half that of object B.
c) The maximum speed of A is the same as that of object B.
d) The maximum speed of A is two times that of object B.
e) The maximum speed of A is four times that of object B.

$V_{max} = \downarrow \omega \, A \uparrow$

$= \text{same}$

$\uparrow T = \dfrac{2\pi}{\omega \downarrow}$



Clicker Question 5:

An 0.80 kg object is attached to one end of a spring, and the system is set into simple harmonic motion. The displacement of *x* of the object as a function of time is shown in the drawing. What is the correct equation for *x*?

$\omega = \frac{2\pi}{T}$

$= \frac{2\pi}{4s} = \frac{\pi}{2}$

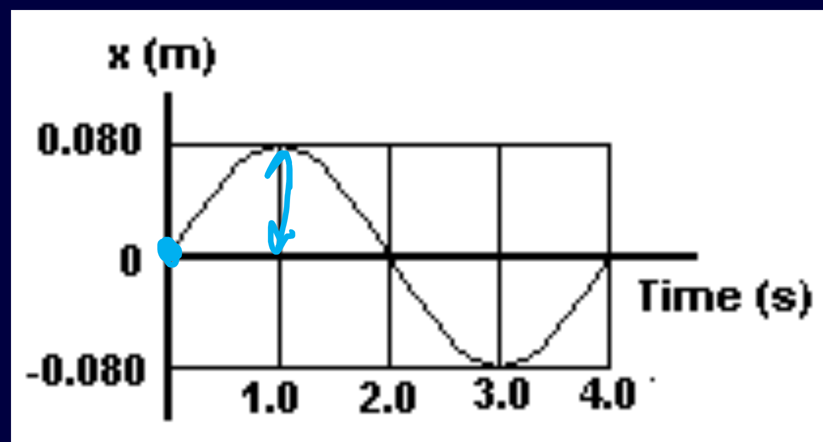
$T = 4s$

(a) $x = 0.080\cos(\pi t)$

(b) $x = 0.160\sin(\pi t)$

(c) $x = 0.080\sin\left(\frac{\pi}{2}t\right)$

(d) $x = 0.1\cancel{6}0\sin\left(\frac{\pi}{2}t\right)$



Simple Harmonic Motion:

x_{max} at $t=0$

$$\boxed{\begin{aligned} x(t) &= [A]\cos(\omega t) \\ v(t) &= -[\omega A]\sin(\omega t) \\ a(t) &= -[\omega^2 A]\cos(\omega t) \end{aligned}}$$

OR

$$\boxed{\begin{aligned} x(t) &= [A]\sin(\omega t) \\ v(t) &= [\omega A]\cos(\omega t) \\ a(t) &= -[\omega^2 A]\sin(\omega t) \end{aligned}}$$

v_{max} $t=0$

$x_{max} = A$
$v_{max} = A\omega$
$a_{max} = A\omega^2$

Period = T (seconds per cycle)
Frequency = f = 1/T (cycles per second)
Angular frequency = $\omega = 2\pi f = 2\pi/T$

phase

$$\sin\left(\underbrace{\omega t + \phi}\right)$$

phase constant

General

$$\boxed{\begin{aligned} x(t) &= A\cos(\omega t + \phi) \\ v(t) &= -\omega A\sin(\omega t + \phi) \\ a(t) &= -\omega^2 A\cos(\omega t + \phi) \end{aligned}}$$

Simple Harmonic Motion:

$x(t) = [A]\cos(\omega t)$

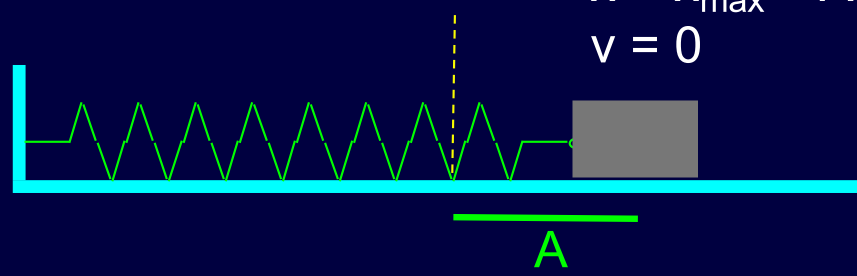
$v(t) = -[A\omega]\sin(\omega t)$

$a(t) = -[A\omega^2]\cos(\omega t)$

$t = 0$

$x = x_{max} = A$

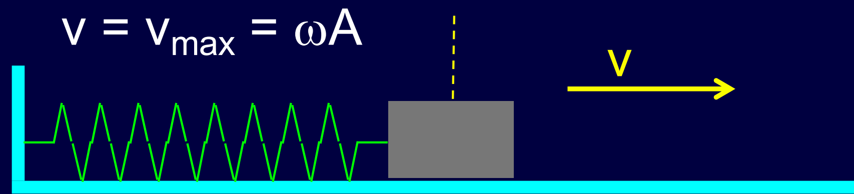
$v = 0$



$t = 0$

$x = 0$

$v = v_{max} = \omega A$



$x(t) = [A]\sin(\omega t)$

$v(t) = [A\omega]\cos(\omega t)$

$a(t) = -[A\omega^2]\sin(\omega t)$

Simple Harmonic Motion:

$x(t) = A\cos(\cancel{\omega}t + \phi)$ ⇨ $x(0) = A\cos(\phi) = A$

t = 0

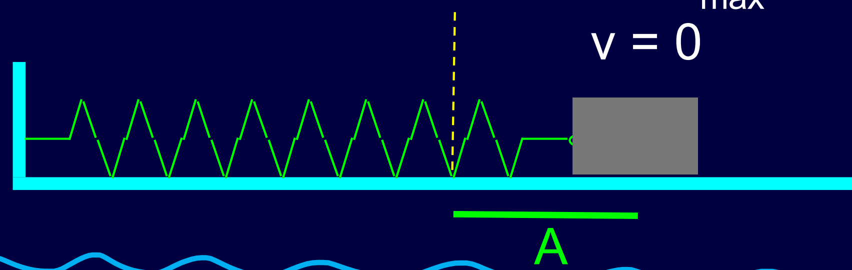
$x = x_{max} = A$

v = 0

$\phi = 0$

$x(t) = A\cos(\omega t)$

$x(0) = A\cos(0) = A$



$x(t) = A\cos(\omega t + \phi)$

t = 0

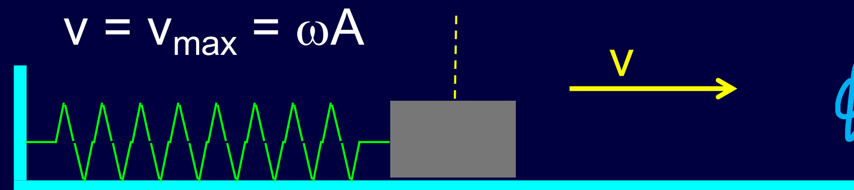
x = 0

$v = v_{max} = \omega A$

$x(0) = A\cos(\phi) = 0$

$\boxed{\phi = -\frac{\pi}{2}}$ ✓

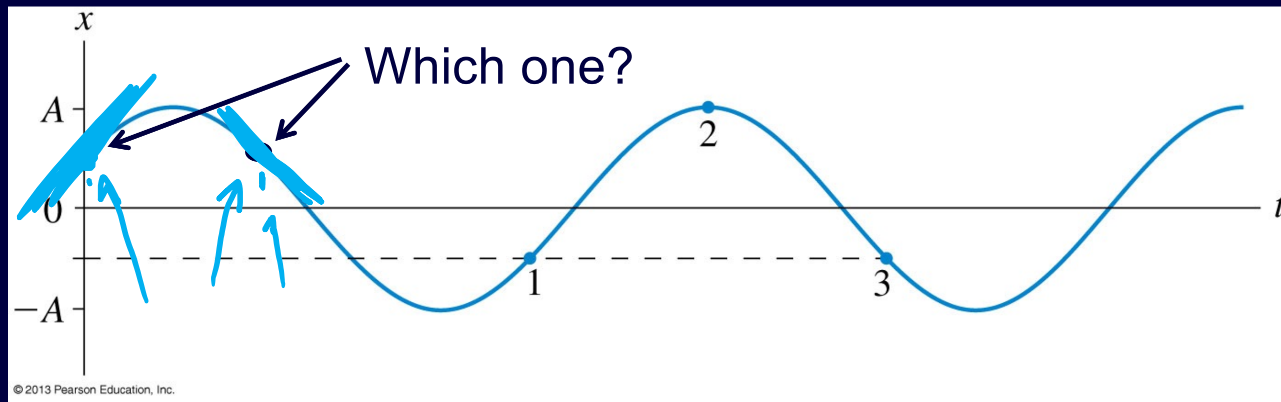
$\phi = \pm\frac{\pi}{2}$



$x(0) = A\cos\left(-\frac{\pi}{2}\right) = 0$

$x(t) = A\cos\left(\omega t - \frac{\pi}{2}\right) = A\sin(\omega t$

Phase Constant



$t = 0$

$x(0) = \frac{A}{2}$

$v(0) = +$

$$x(t) = A\cos(\cancel{\omega t} + \phi)$$

$$v(t) = -\omega A\sin(\cancel{\omega t} + \phi)$$

$$x(0) = \cancel{A}\cos(\phi) = \frac{\cancel{A}}{2}$$

$$v(0) = -\omega A\sin(\phi)$$

$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

$\cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$

$$\cos(\phi) = \frac{1}{2}$$

$$v(0) = -\omega A\sin\left(-\frac{\pi}{3}\right) = \text{positive}$$

✓

$$\phi = \pm 1.047 = \pm\frac{\pi}{3}$$

$$\boxed{x(t) = A\cos\left(\omega t - \frac{\pi}{3}\right)}$$

✓

Period for a Mass on a Spring

- What can Newton's second law tell us about SHM?

y, x

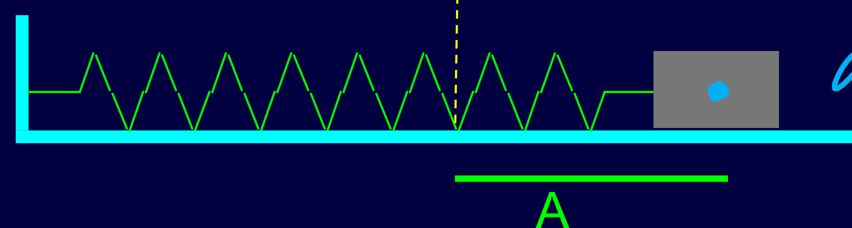
$x(t)$

differential equation

$$\sum F = ma$$

$$-kx = ma$$

$$-kx = m\frac{d^2x}{dt^2}$$



$v = \frac{dx}{dt}$

$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$x(t) = A\cos(\omega t)$$

$$\dot{x}(t) = -\omega A\sin(\omega t)$$

$$\ddot{x}(t) = -\omega^2 A\cos(\omega t)$$

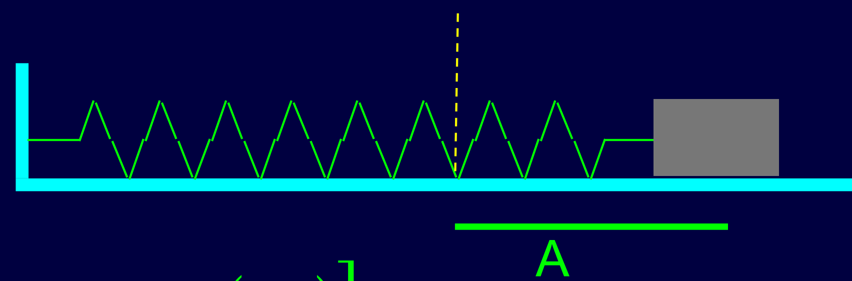
$$\boxed{-\omega^2}\,\cancel{A\cos\omega t} = -\frac{k}{m}\cancel{A\cos\omega t}$$

$$\omega^2 = \frac{k}{m} \Rightarrow \omega = \sqrt{\frac{k}{m}}$$

Period for a Mass on a Spring

- What can Newton's second law tell us about SHM?

$$\sum F = ma$$



$$-kx = ma$$

$$-k[\underbrace{A\cos(\omega t)}_{x}] = m[-\underset{\frac{d^2x}{dt^2}}{A\omega^2\cos(\omega t)}]$$

$$k = m\omega^2$$

$$\rightarrow \boxed{\omega = \sqrt{\frac{k}{m}}} \qquad \therefore \boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

$w = \sqrt{\frac{k}{m}}$

Clicker Question 6:

A mass is attached to a spring. I pull it distance of A and it oscillates with frequency f. If I pull it a distance of 2A what will the frequency be?

$w = 2\pi f$

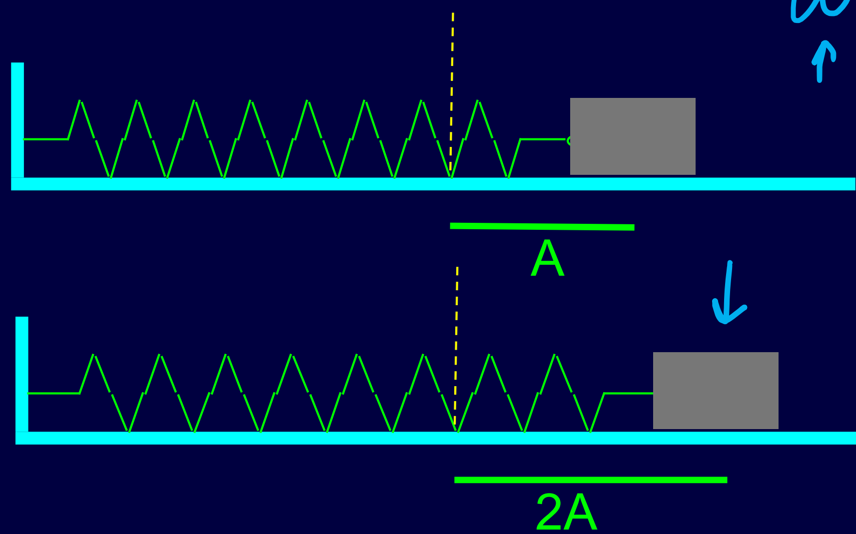
(a) 4f

(b) 2f

(c) f

(d) f/2

(e) f/4



$$\omega = \sqrt{\frac{k}{m}}$$

Demo

Clicker Question 7:

$\downarrow \omega = \sqrt{\frac{k}{m}} \uparrow$

A block of mass m oscillates on a horizontal spring with period $T = 2.0$ s. If a second identical block is glued to the top of the first block, the new period will be

$\uparrow T = \frac{2\pi}{\omega} \downarrow$

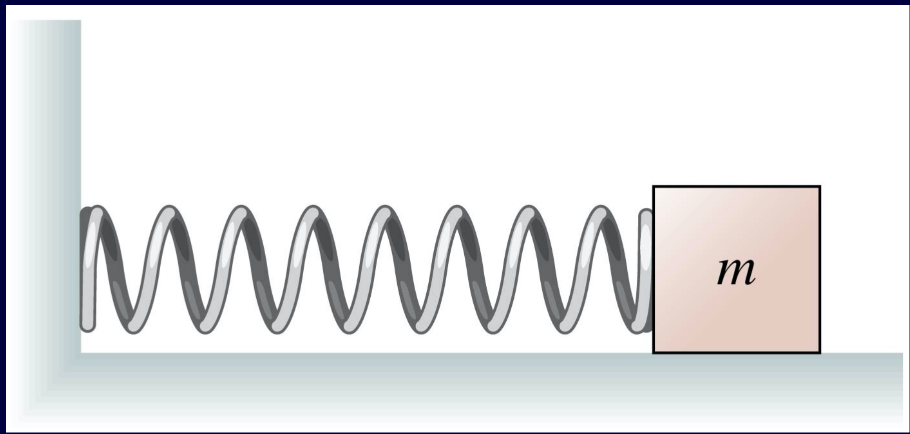
A. 1.0 s.

B. 1.4 s.

C. 2.0 s.

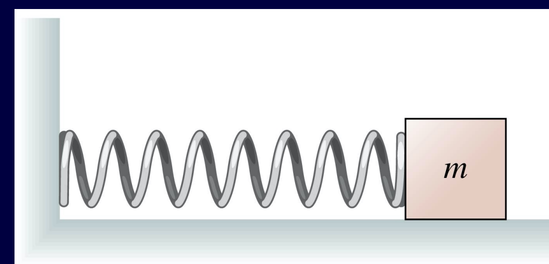
D. 2.8 s.

E. 4.0 s.



Clicker Question 8:

A block of mass m oscillates on a horizontal spring with period $T = 2.0$ s. If a second identical block is glued to the top of the first block, the new period will be



$$T = \frac{2\pi}{\omega}$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$\mathrm{T_i} = 2\pi\sqrt{\frac{m_i}{k}}$$

$$m_f = 2m_i$$

$$\mathrm{T_f} = 2\pi\sqrt{\frac{m_f}{k}}$$

$$\frac{T_f}{T_i} = \frac{2\pi\sqrt{\frac{m_f}{k}}}{2\pi\sqrt{\frac{m_i}{k}}}$$

$$\frac{\mathrm{T_f}}{\mathrm{T_i}} = \sqrt{\frac{m_f}{m_i}}$$

$$\frac{\mathrm{T_f}}{\mathrm{T_i}} = \sqrt{\frac{2m}{m}}$$

$$\mathrm{T_f} = \sqrt{2}\,\mathrm{T_i}$$

$$= 2.8\ \mathrm{s}$$