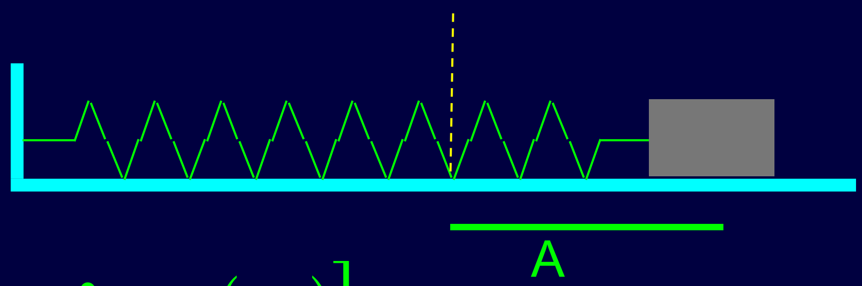
Period for a Mass on a Spring

- What can Newton's second law tell us about SHM?

$$\sum F = ma$$

$$-kx = ma$$



$$-k\left[A\cos(\omega t)\right] = m\left[-A\omega^2\cos(\omega t)\right]$$

$$k = m\omega^2$$

$$\boxed{\omega = \sqrt{\frac{k}{m}}} \qquad \therefore \boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

Clicker Question 0.5:

A mass is attached to a spring. I pull it distance of A and it oscillates with frequency f. If I pull it a distance of 2A what will the frequency be?

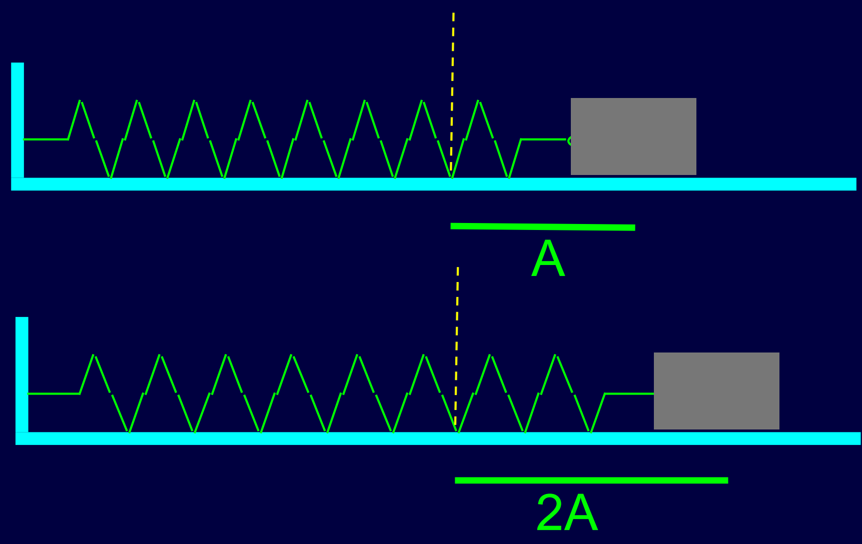
(a) 4f

(b) 2f

(c) f

(d) f/2

(e) f/4



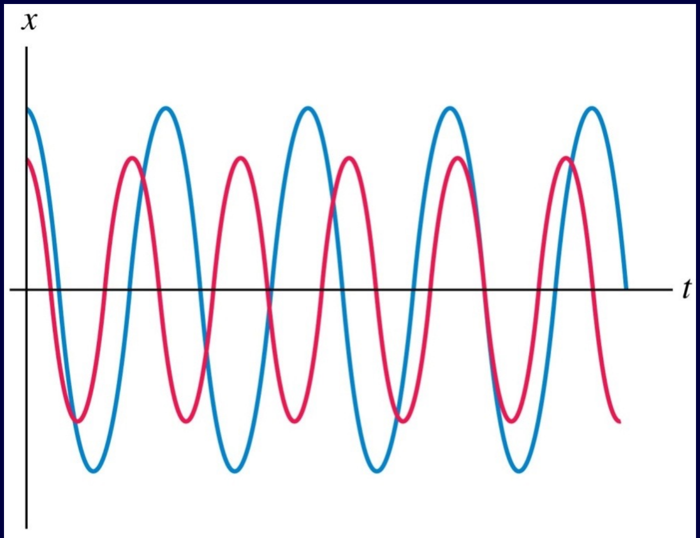
Clicker Question 0.6:

Two identical blocks oscillate on different horizontal springs. Which spring has the larger spring constant?

A. The red spring.

B. The blue spring.

C. There's not enough information to tell.



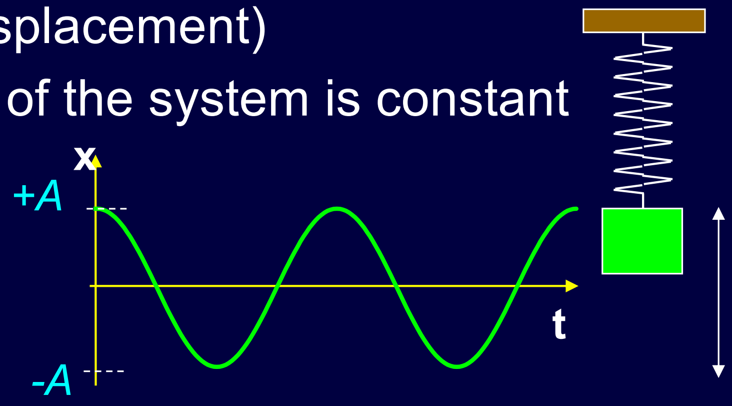
Clicker Question 1:

A mass on a spring oscillates back & forth with simple harmonic motion of amplitude *A*. A plot of displacement (x) versus time (t) is shown below. At what points during its oscillation is the total mechanical energy (K+U) of the mass and spring a maximum? (Ignore friction).

a) When x = +A or -A (i.e. maximum displacement)

b) When x = 0 (i.e. zero displacement)

c) The mechanical energy of the system is constant



Energy Conservation

- If there are no non-conservative forces acting, the mechanical energy will be conserved:

$$E = K + U = \tfrac{1}{2} m v^2 + \tfrac{1}{2} k x^2$$

- At maximum displacement, x=*A*, v = 0:

$$E = \tfrac{1}{2} k A^2$$

$$v_{\max} = \omega A$$

- At zero displacement, x = 0:

$$E = \tfrac{1}{2} m {v_{\max}}^2 = \tfrac{1}{2} m \omega^2 A^2$$

$$E = \tfrac{1}{2} m \frac{k}{m} A^2 = \tfrac{1}{2} k A^2$$

$$\omega^2 = \frac{\mathrm{k}}{\mathrm{m}}$$

Energy Conservation

- This may also be shown more formally:

$$E = \tfrac{1}{2} m v^2 + \tfrac{1}{2} k x^2$$

- Find this as a function of time:

$$E = \tfrac{1}{2} m\left[-\omega A \sin(\omega t)\right]^2 + \tfrac{1}{2} k\left[A \cos(\omega t)\right]^2$$

$$E = \tfrac{1}{2} m \omega^2 A^2 \sin^2(\omega t) + \tfrac{1}{2} k A^2 \cos^2(\omega t)$$

$$E = \tfrac{1}{2} k A^2 \sin^2(\omega t) + \tfrac{1}{2} k A^2 \cos^2(\omega t)$$

$$E = \tfrac{1}{2} k A^2\left(\sin^2(\omega t) + \cos^2(\omega t)\right) = \tfrac{1}{2} k A^2$$

Clicker Question 2:

A mass oscillates in simple harmonic motion with amplitude A. If the mass is doubled, but the amplitude is not changed, what will happen to the total mechanical energy of the system?

a) total energy will increase

b) total energy will not change

c) total energy will decrease

Example

A block of mass 5.0 kg on a frictionless surface is attached to a horizontal spring with k = 32 N/m. If the block is pulled 0.20 m from the equilibrium position of the spring, what velocity will it have when it is 0.10 m from the equilibrium position?

$$K_i + U_i = K_f + U_f$$

$$\cancel{\tfrac{1}{2} m v_i{}^2} + \tfrac{1}{2} k x_i{}^2 = \tfrac{1}{2} m v_f{}^2 + \tfrac{1}{2} k x_f{}^2$$

$$v_f = \sqrt{\frac{k}{m}\left(x_i^2 - x_f^2\right)} \qquad = 0.44\,\mathrm{m/s}$$

$$K_i + U_i = \tfrac{1}{2} 32\mathrm{N/m}\left(0.20\mathrm{m}\right)^2 = 0.64\,\mathrm{J}$$

$$K_f + U_f = \tfrac{1}{2} 32\mathrm{N/m}\left(0.10\mathrm{m}\right)^2 + \tfrac{1}{2} 5\mathrm{kg}\left(0.44\mathrm{m/s}\right)^2 = 0.64\,\mathrm{J}$$

SHM
Clicker Question 3

What approximation is necessary in order for a pendulum to execute simple harmonic motion? Answer all that apply.

A. No approximations are necessary; a pendulum always executes simple harmonic motion.
B. The mass must be very large.
C. The mass must be very small.
D. The length must be very large.
E. The amplitude of the angular displacement must be less than 10 degrees.

Pendulum Motion

- Free-body diagram for pendulum:

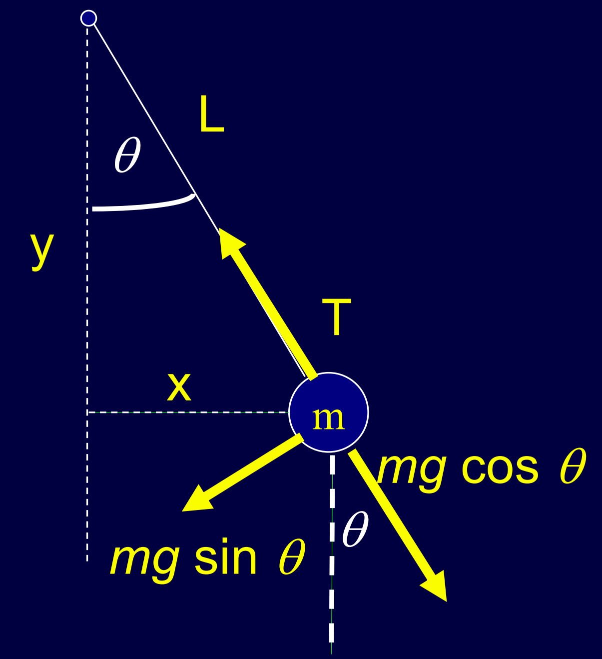
$$F_{\tan} = mg\sin\theta$$

For small oscillations

$$F_{\tan} \approx mg\theta$$

$$F_{\tan} = mg\frac{s}{L} = \frac{mg}{L}s$$

$$F = kx \qquad k = \frac{mg}{L}$$

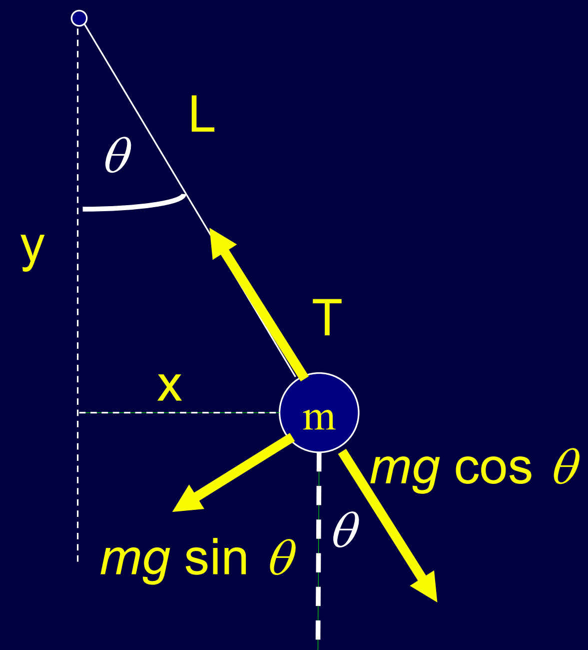


Pendulum Motion

- Free-body diagram for pendulum:

$$k = \frac{mg}{L} \qquad \omega = \sqrt{\frac{\mathrm{k}}{\mathrm{m}}} = \sqrt{\frac{\mathrm{g}}{\mathrm{L}}}$$

$$\mathrm{T} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{L}{g}}$$



Period and frequency do not depend on A, or m!
(For small oscillations)

Clicker 4:

On Planet X, a ball on a massless, rigid rod oscillates as a simple pendulum with a period of 2.0 s. If the pendulum is taken to the moon of Planet X, where the free-fall acceleration *g* is half as big, the period will be

A. 1.0 s

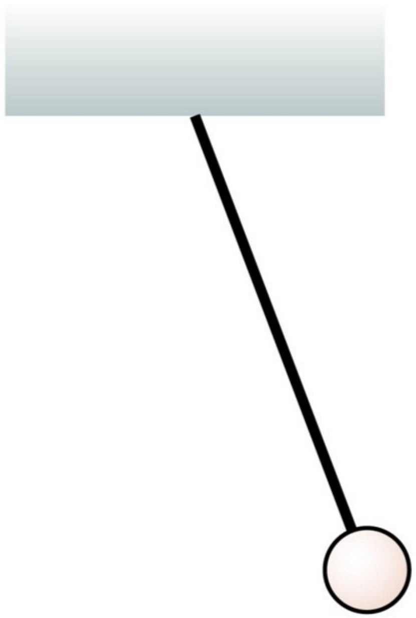
B. 1.4 s

C. 2.0 s

D. 2.8 s

E. 4.0 s

$$T \propto \frac{1}{\sqrt{g}}$$



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$$\mathrm{T} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{L}{g}}$$

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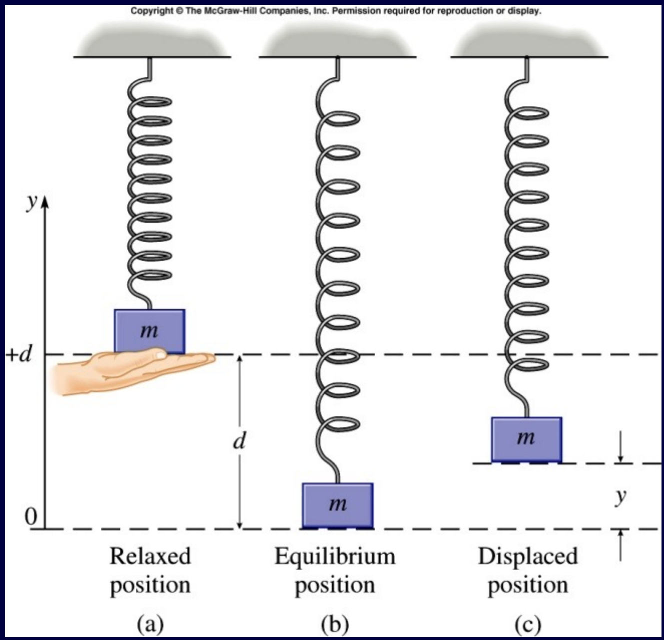
Vertical Mass and Spring

If we include gravity, there are two forces acting on mass. With mass, new equilibrium position has spring stretched d.



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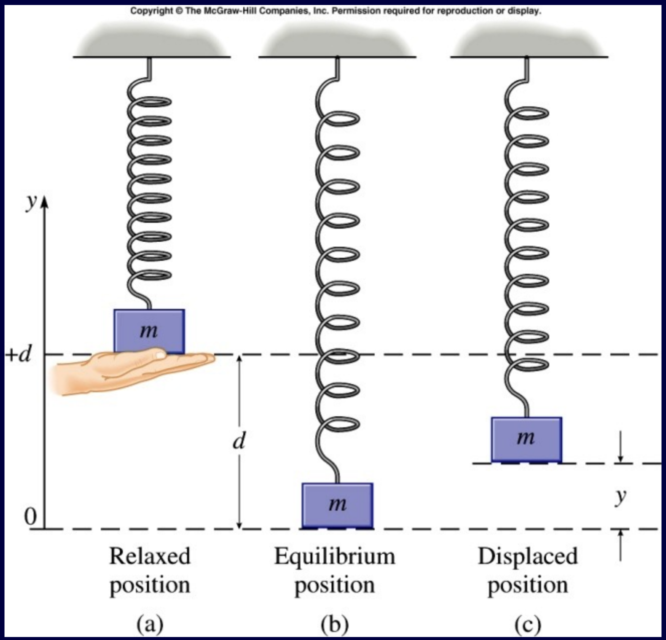


Damped Oscillations (1 of 4)

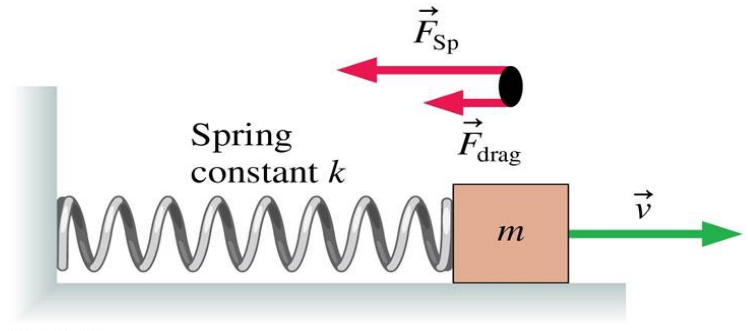
- An oscillation that runs down and stops is called a damped oscillation.
- The shock absorbers in cars and trucks are heavily damped springs.
- The vehicle's vertical motion, after hitting a rock or a pothole, is a damped oscillation.
- One possible reason for dissipation of energy is the drag force due to air resistance.

- The forces involved in dissipation are complex, but a simple **linear drag** model is

$$\vec{F}_{\text{drag}} = -b\vec{v} \quad \text{(model of the drag force)}$$

Damped Oscillations (2 of 4)

- When a mass on a spring experiences the force of the spring as given by Hooke's Law, as well as a linear drag force of magnitude



$|F_{\text{drag}}| = bv$, the solution is

$$x(t) = Ae^{-bt/2m}\cos(\omega t + \phi_0) \quad \text{(damped oscillator)}$$

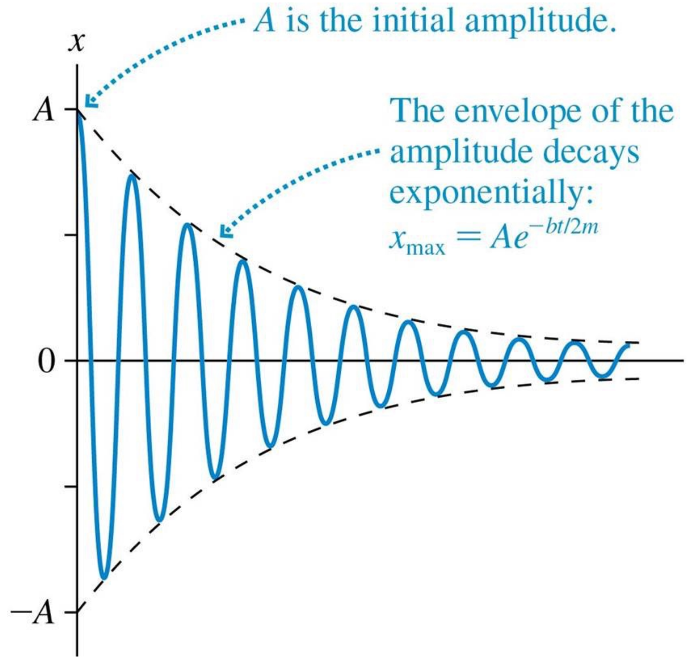
where the angular frequency is given by

$$\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = \sqrt{\omega_0{}^2 - \frac{b^2}{4m^2}}$$

- Here $\omega_0 = \sqrt{k/m}$ is the angular frequency of the undamped oscillator (b = 0).

Damped Oscillations (3 of 4)

- Position-versus-time graph for a damped oscillator.

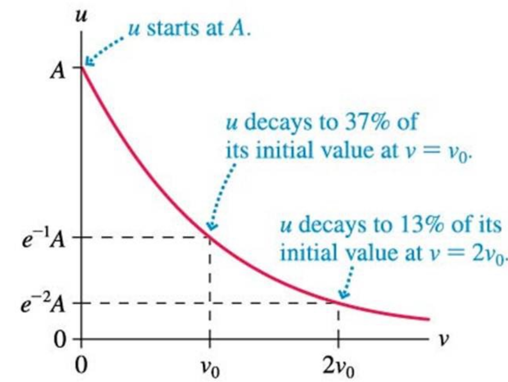


Mathematical Aside: Exponential Decay

- **Exponential decay** occurs in a vast number of physical systems of importance in science and engineering.
- Mechanical vibrations, electric circuits, and nuclear radioactivity all exhibit exponential decay.
- The graph shows the function:

  $$u = Ae^{-v/v_0} = A\exp\left(-v/v_0\right)$$

  where
- e = 2.71828… is Euler's number.
- exp is the **exponential function.**
- v_0 is called the **decay constant.**

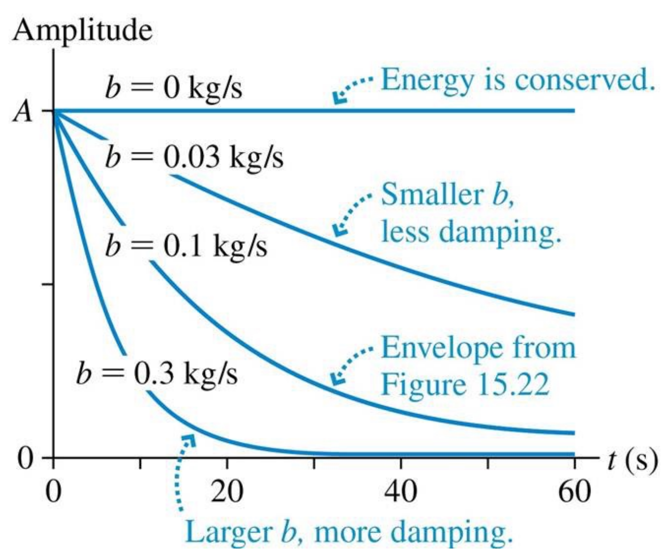


Damped Oscillations (4 of 4)

- A damped oscillator has position $x = x_{max}\cos(\omega t + \phi_0)$, where

$$x_{max}(t) = Ae^{-bt/2m}$$

- This slowly changing function x_{max} provides a border to the rapid oscillations, and is called the **envelope.**
- The figure shows several oscillation envelopes, corresponding to different values of the damping constant *b*.



Energy in Damped Systems

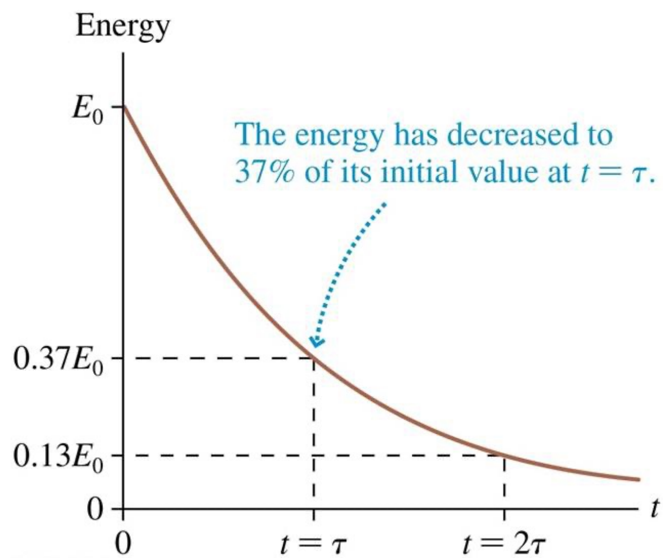
- Because of the drag force, the mechanical energy of a damped system is no longer conserved.
- At any particular time we can compute the mechanical energy from

$$E(t) = E_0 e^{-t/\tau}$$

- Where the decay constant of this function is called the **time constant**

τ, defined as $\tau = \dfrac{m}{b}$

- The oscillator's mechanical energy decays exponentially with time constant τ.



Pearson

Clicker Question 5

During a time interval of one time constant, by what factor does the mechanical energy of an oscillating system decrease?

A. By a factor of ln(2) (0.6931)
B. By a factor of 2
C. By a factor of e (2.718)
D. By a factor of pi
E. By a factor of 10

Clicker Question 6:

Here are three sets of values for spring constant, damping constant, and mass for the damped oscillator. Which will take the greatest time for the mechanical energy to decrease to ¼ of its original value?

a) $2k_0$, b_0, and m_0

b) k_0, $6b_0$, and $4m_0$

c) $3k_0$, $3b_0$, and m_0

Clicker Question 7:

The amplitude of an oscillator decreases to 36.8% of its initial value in 10 s. What is the value of its time constant?

$$x(t) = Ae^{-bt/2m}\cos(\omega t + \phi_0) \quad (\text{damped oscillator})$$

a) 0.368 s

b) 1 s

c) 10 s

d) 5 s

e) 20 s

$$\tau = \frac{m}{b}$$

Driven Oscillations and Resonance (1 of 3)

- Consider an oscillating system that, when left to itself, oscillates at a **natural frequency** f_0.
- Suppose that this system is subjected to a **periodic** external force of **driving frequency** f_{ext}.
- The amplitude of oscillations is generally not very high if f_{ext} differs much from f_0.
- As f_{ext} gets closer and closer to f_0, the amplitude of the oscillation rises dramatically.
- A singer or musical instrument can shatter a crystal goblet by matching the goblet's natural oscillation frequency.

Driven Oscillations and Resonance (2 of 3)

- The response curve shows the amplitude of a driven oscillator at frequencies near its natural frequency of 2.0 Hz.