# Lecture 8: Work and Energy

## Today's Agenda

- Work and Energy
  - Definition of work
  - Examples
- Definition of Mechanical Energy
- Conservation of Mechanical Energy
  - Conservative forces

# Start Recording

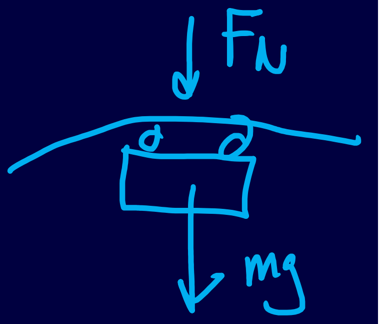
# Clicker Question 0.1:

A roller coaster car is at the top of a loop-de-loop and does not lose contact with the track. The loop has a diameter of 20m.

How many forces act on the car?

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5

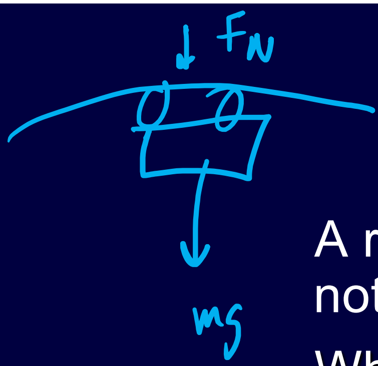

(a)



A roller coaster car is at the top of a loop-de-loop and does not lose contact with the track.

How many forces act on the car?

(a)




# Clicker Question 0.2:

A roller coaster car is at the top of a loop-de-loop and does not lose contact with the track.

What force provides the centripetal acceleration required to keep the car moving in a circle?

(a) Normal Force

(b) Gravity

(c) Both (a) and (b)

(a)

What is the minimum speed you must have at the top of the roller coaster loop to keep the wheels on the track? The loop has a diameter of 20m.

+y ↓

$F_N$ ↓, $mg$ ↓

$$\Sigma F_y = ma_y$$

$$\cancel{F_N} + mg = m\frac{v^2}{r}$$

$$\cancel{m}g = \cancel{m}\frac{v^2}{r}$$

$r = 10\ m$

$$v_c = \sqrt{g r}$$

$=$

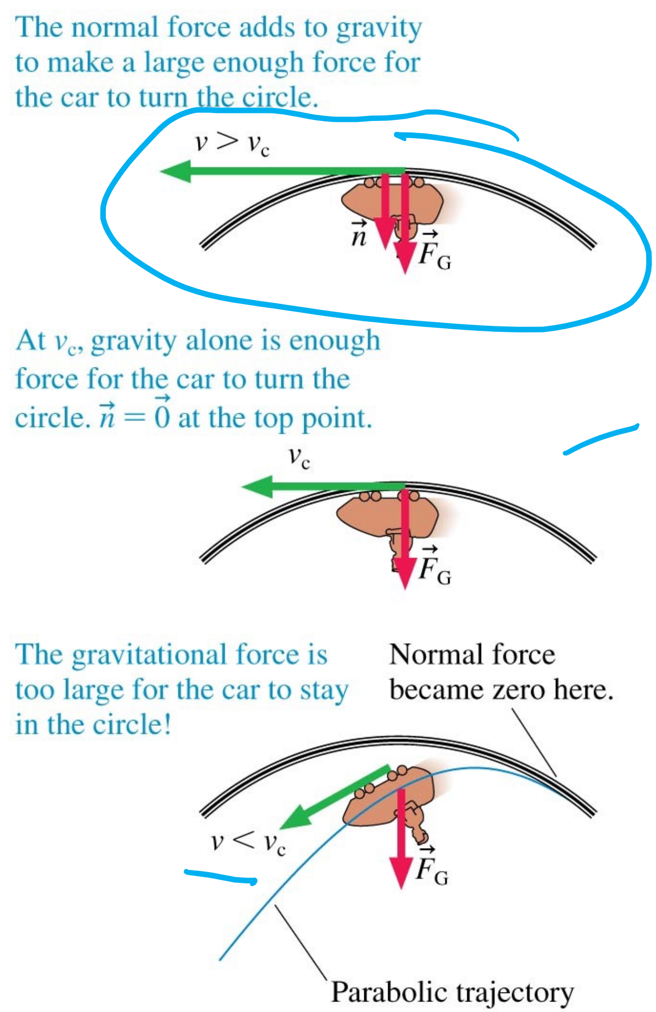
$$v_c = \sqrt{gR} = 9.9\ m/s$$

# Loop-the-Loop

A roller-coaster car at the top of the loop.

$v_c$

$\uparrow F_N + mg = \frac{mv^2}{r} \uparrow$



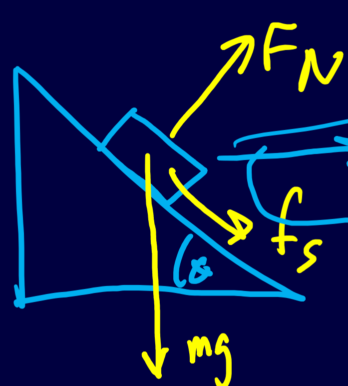

$-mg = \frac{mv^2}{r}$

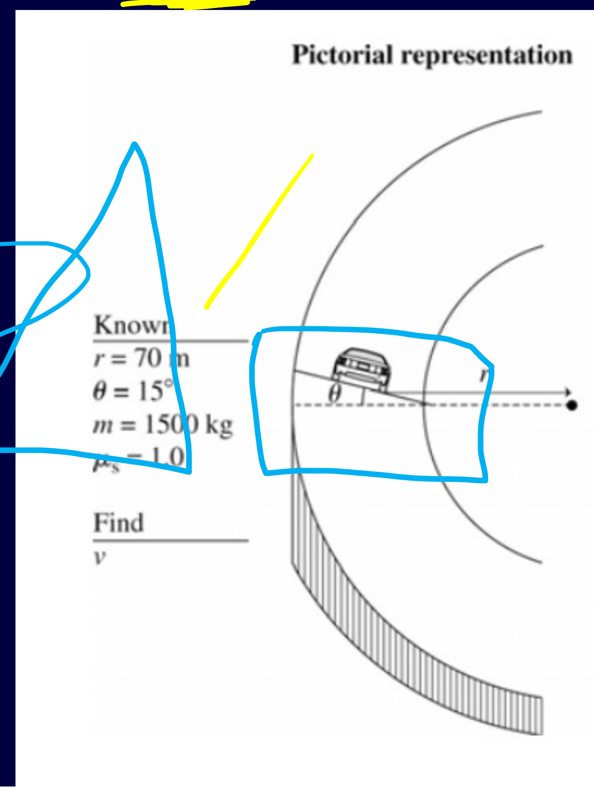
# Clicker Question 4:

A concrete highway curve of radius r = 70 m is banked at a a angle of 15 °. What is the maximum speed with which a 1500 kg rubber-tired car can take this curve without sliding? (Take the static coefficient of friction of rubber on concrete to be 1.0.)

How many forces act on the car?

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

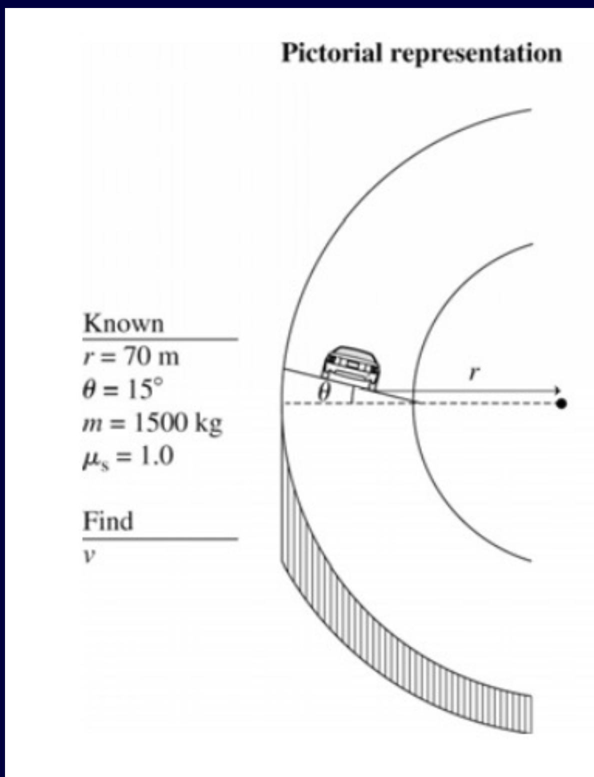


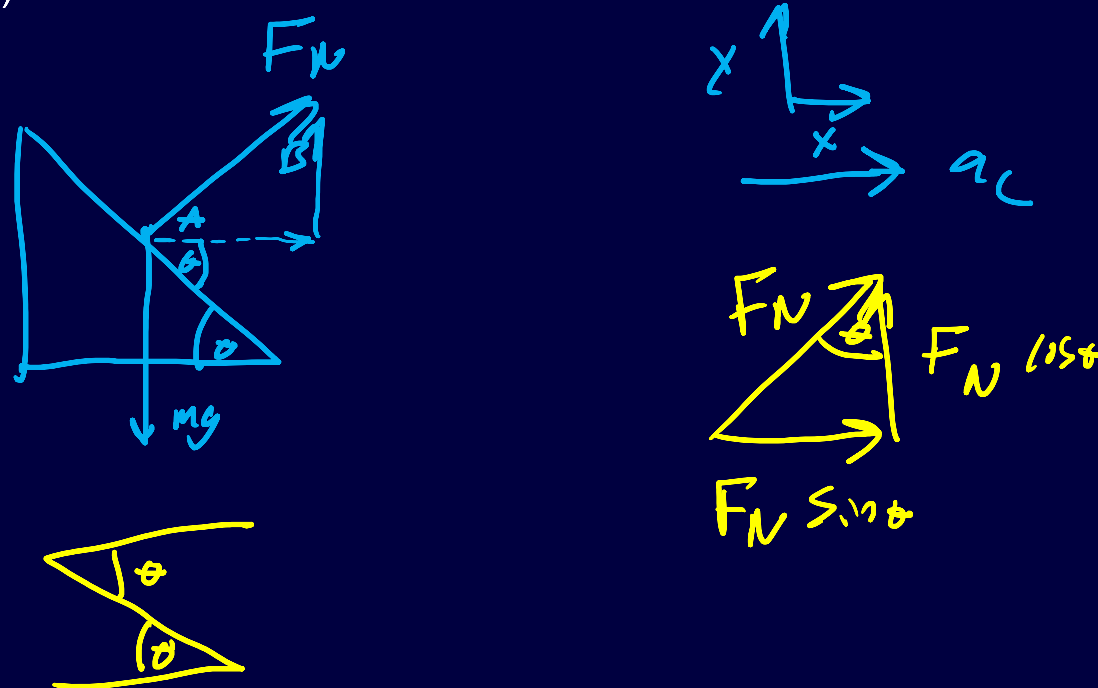



# Example 8.40

$A + B = 90$ $A + \theta = 90$

A concrete highway curve of radius r = 70 m is banked at a a angle of 15 °. What is the maximum speed with which a 1500 kg rubber-tired car can take this curve without sliding? (Take the static coefficient of friction of rubber on concrete to be 1.0.)



# Example 8.40

A concrete highway curve of radius r = 70 m is banked at a a angle of 15 °. What is the maximum speed with which a 1500 kg rubber-tired car can take this curve without sliding? (Take the static coefficient of friction of rubber on concrete to be 1.0.)

$$\sum F_y = 0 \qquad F_s^{max} = \mu F_N \qquad \sum F_x = m a_c$$

$$F_N \cos\theta - F_s \sin\theta - mg = 0 \qquad F_N \sin\theta + F_s \cos\theta = \frac{mv^2}{r}$$

$$F_N \cos\theta - \mu F_N \sin\theta = mg \qquad F_N \sin\theta + \mu F_N \cos\theta$$

$$F_N(\cos\theta - \mu \sin\theta) = mg \qquad F_N(\sin\theta + \mu\cos\theta) = \frac{mv^2}{r}$$

$$F_N = \frac{mg}{\cos\theta - \mu\sin\theta} \qquad \frac{mg}{\cos\theta - \mu\sin\theta}(\sin\theta + \mu\cos\theta) = \frac{mv^2}{r}$$

# Example 8.40

A concrete highway curve of radius r = 70 m is banked at a a angle of 15 °. What is the maximum speed with which a 1500 kg rubber-tired car can take this curve without sliding? (Take the static coefficient of friction of rubber on concrete to be 1.0.)

$$g\frac{(\sin\theta + \mu\cos\theta)}{\cos\theta - \mu\sin\theta} = \frac{v^2}{r}$$

$$v = \sqrt{\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right) g r}$$

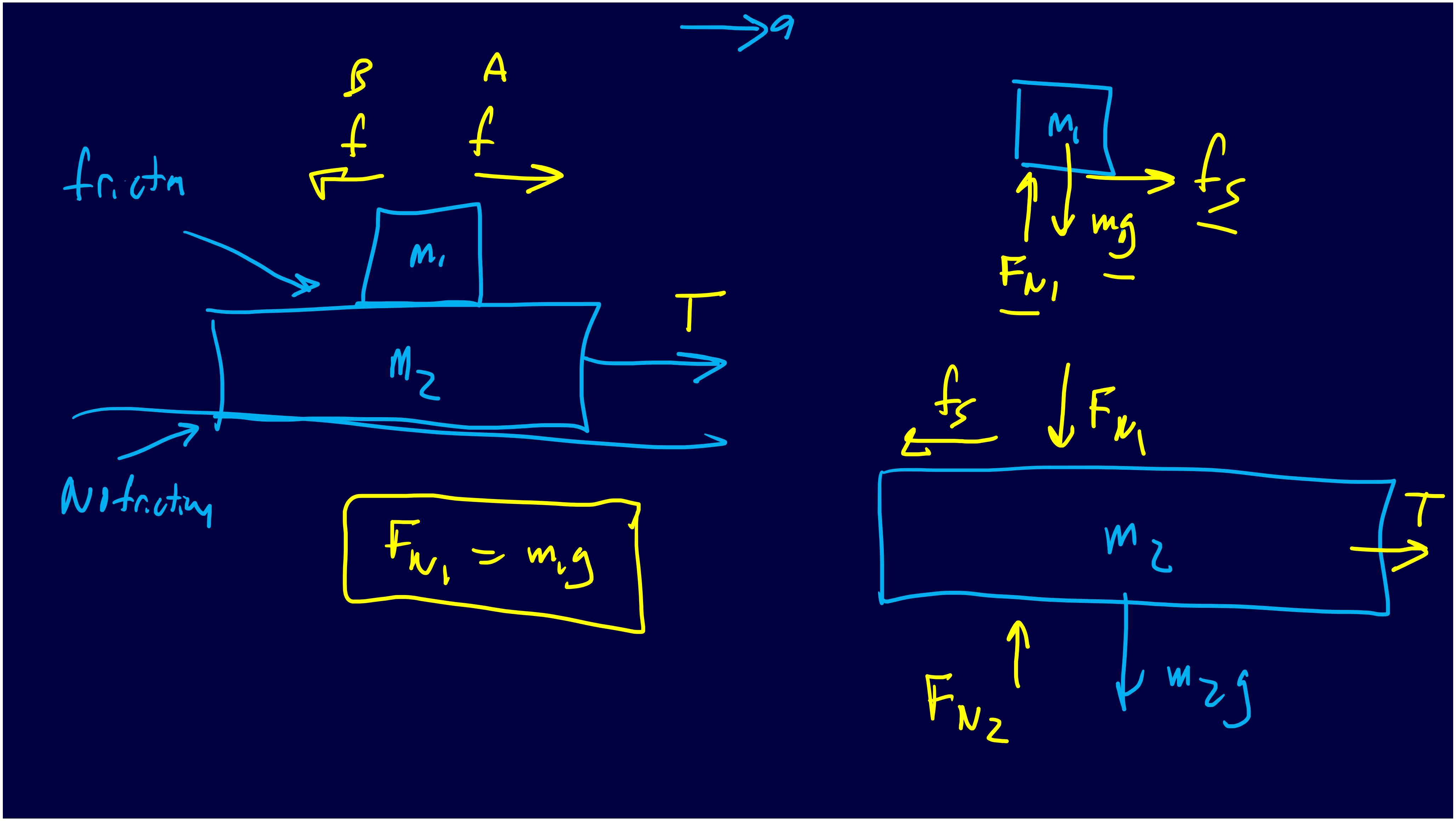
$$= 34.5\ m/s$$

$\mu = 1$

$\theta = 15$

$r = 70m$

$g = 9.81\ m/s^2$


a
B
A
f
f
friction
m1
m2
T
No friction
FN1 = m1g
m1
fs
mg
FN1
fs
FN1
m2
T
FN2
m2g

## New section: Why some things don’t change

If something doesn’t change we say it is conserved

Concepts like Energy and Momentum are less tangible than mass and force

They will give us a new and different perspective on motion

# Work by a Constant Force

- Work is the energy transferred to a system or object that is caused by a force

- To do work you need two things
  - A Force
  - Motion

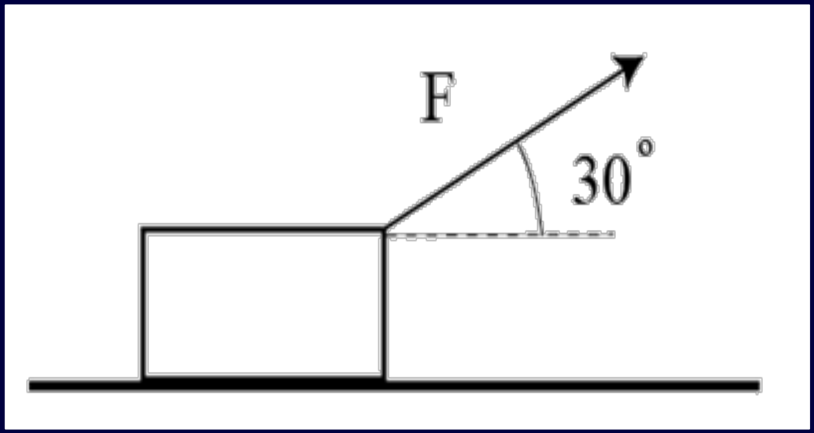
- I lift my book with force F through a distance of $\Delta s$
- Work = $F \times \Delta s$ = Force × (Displacement)
  - Equation only works with a constant force
  - Equation only works when the force is in same direction as displacement

$$\text{Work} = \text{N} \times \text{m} = \text{Joule} = \text{J}$$

# Clicker Question 1:

A box sits on the floor. A force F = 200 N is then applied to the right but slightly upward at an angle of 30° from the horizontal (see diagram) such that the box moves 5 meters horizontally to the right. How much work is done by the force?

A. 50 J
B. 866 J
C. 1900 J
D. 2430 J
E. 3230 J




Cos 30 = 0.866
Sin 30 = 0.5

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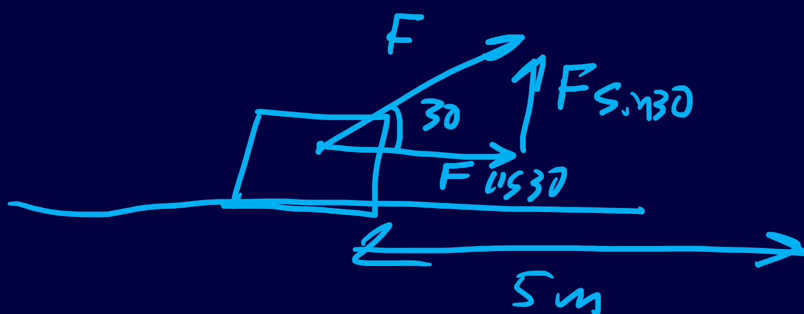
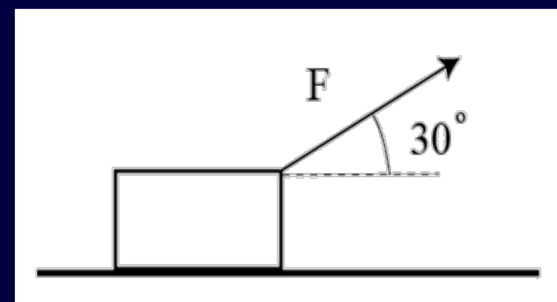



Cos 30 = 0.866
Sin 30 = 0.5

Work = F Δs cos(θ)

$W = F_{\parallel} d$

$= F \cos 30 \; 5m = 866 J$

F is magnitude of Force;
Δs is displacement
θ is angle between the two

# Interesting things about work

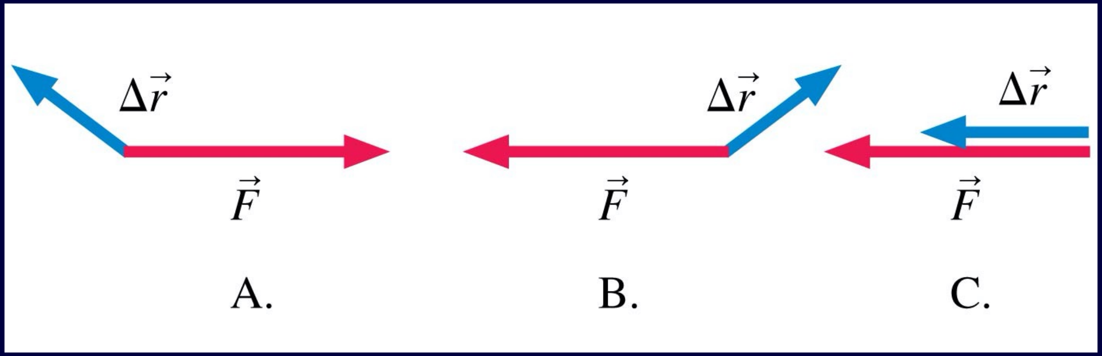
- You have to move the object to DO work!!
  - I hold up a dumbbell, but it doesn’t move
    - I do no work!!
  - Definition of work!
- If force is perpendicular to motion you do no work!
- You can have positive or negative work
  - Work is a scalar (easier for us!)

# Positive vs. negative work

- If component of force points in same direction as displacement, work is positive
- If component of force points in opposite direction as displacement, work is negative
- Good example: Friction (often does negative work)

# Clicker Question 2:

A constant force pushes a particle through a displacement. In which of these three cases does the force do negative work?




D. Both A and B.

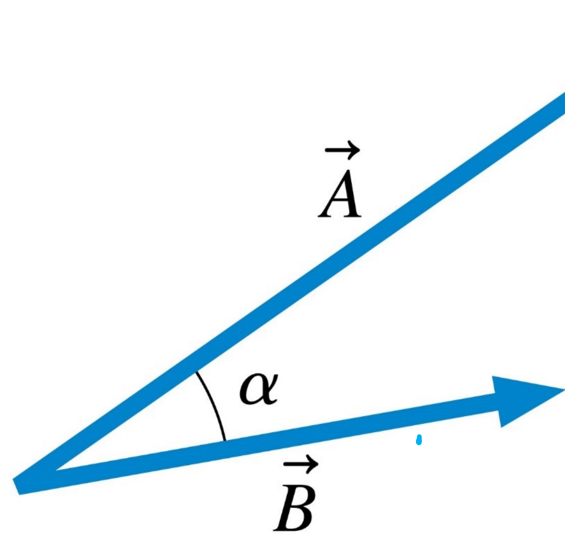
E. Both A and C.

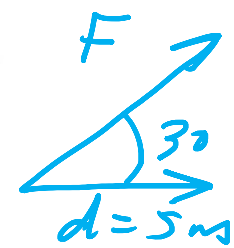
# The Dot Product of Two Vectors

- The figure shows two vectors, $\vec{A}$ and $\vec{B}$, with angle $\alpha$ between them.
- The dot product of $\vec{A}$ and $\vec{B}$ is defined as

$$\vec{A} \cdot \vec{B} = AB\cos\alpha$$

- The dot product is also called the **scalar product** because the value is a scalar.




$W = \vec{F} \cdot \vec{d}$

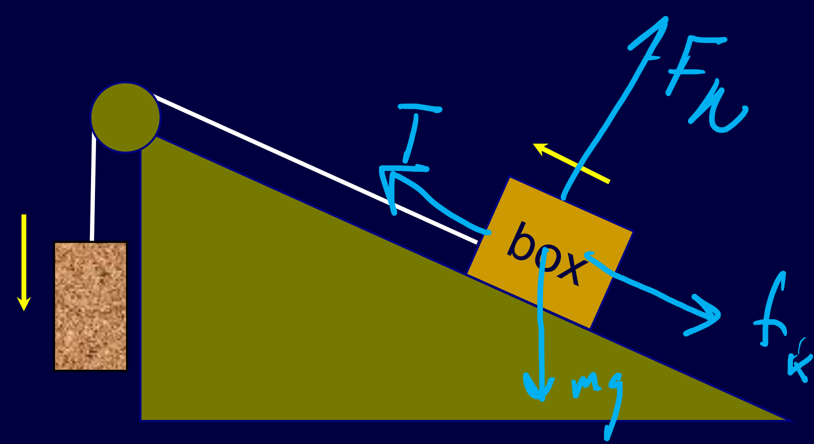
$= |F||d| \cos\alpha$

$= |F| |d| \cos 30$

# Clicker Question 3:

- A box is pulled up a rough ($\mu > 0$) incline by a rope-pulley-weight arrangement as shown below.
  - How many forces are doing work on the box?

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5



# Clicker Question 3:

- Draw FBD of box:
- Consider direction of motion of the box
- Any force not perpendicular to the motion will do work:

$F_N$ does no work (perp. to v)

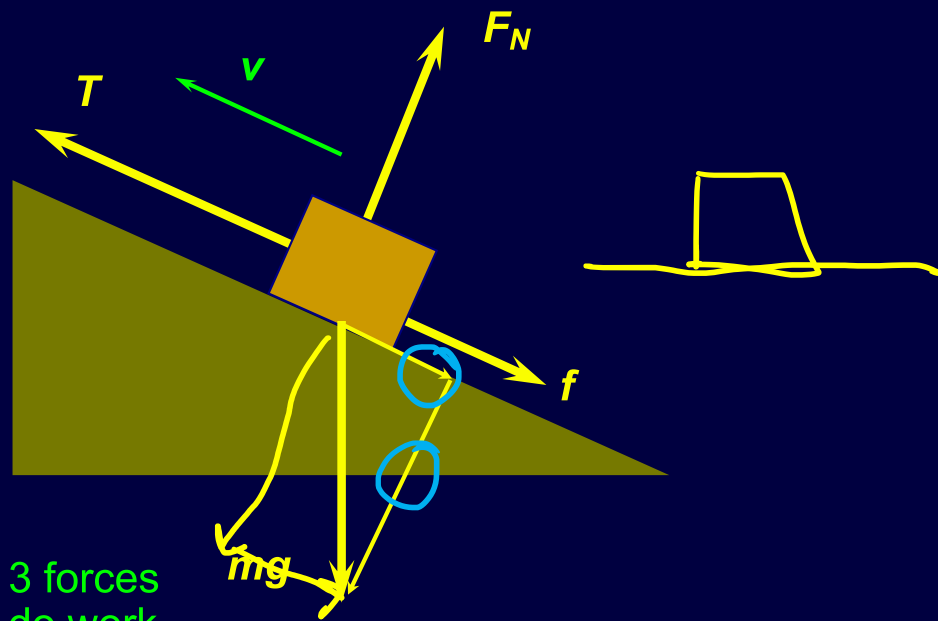
*T* does positive work

*f* does negative work

*mg* does negative work

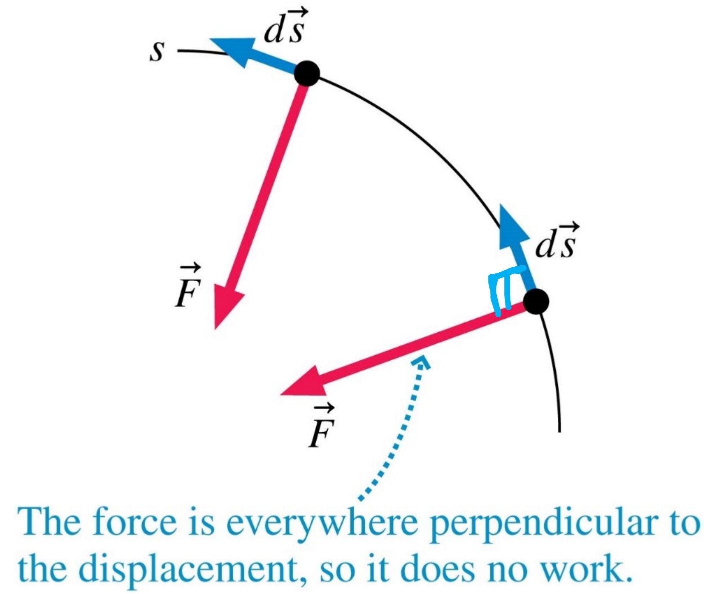
3 forces do work

3 total work



# Zero-Work Situations

- The figure shows a particle moving in uniform circular motion.
- At every point in the motion, $F_s$, the component of the force parallel to the instantaneous displacement, is zero.




- The particle's speed, and hence its kinetic energy, doesn't change, so $W = \Delta K = 0$.
- **A force everywhere perpendicular to the motion does no work.**

## Work Total

- Work total is the work on an object by all forces acting.

$$W_{tot} = \Sigma W$$

- Work total is the work done by the net force

# Kinetic Energy: Motion

- Apply constant force along x-axis to a point particle *m.*

  $W = F_x \Delta s$

  $= m\, a_x \Delta s$

  $= ½\, m\, (v_f^2 - v_i^2)$

$$\text{recall}: a_x \Delta s = \frac{1}{2}(v_{xf}^2 - v_{xi}^2)$$

- Work changes ½ m $v^2$

- This is the Kinetic Energy $K = ½\, m\, v^2$

$W = \Delta K$ For Point Particles

# Kinetic Energy

- An object in motion has energy
- Energy of motion is called Kinetic energy
- True for any object with velocity v and mass m.

$$K = \frac{1}{2} m v^2$$

Joule

# The Work-Energy Theorem

- $W_{tot} = ½mv_f^2 - ½mv_i^2$

$\vec{F} = m\vec{a}$

- Put positive work into an object, its speed increases!
- Put negative work into an object, its speed decreases!
- Be careful, we are interested in total work done!!!
  - We must look at **all** forces acting, not just one!

## Clicker Question 4:

Two identical arrows, one with twice the speed of the other, are fired into a bale of hay. Assuming the hay exerts a constant frictional force on the arrows, the faster arrow will penetrate how much further than the slower arrow?

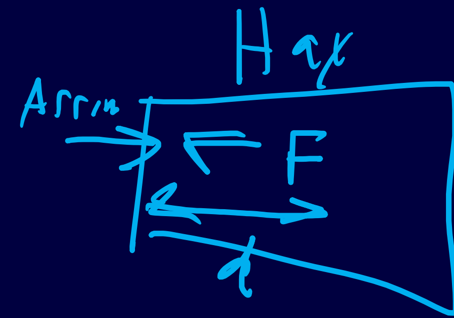
(A) twice as much as the slower arrow

(B) four times as much the slower arrow

(C) six times as much the slower arrow

(D) eight times as much the slower arrow

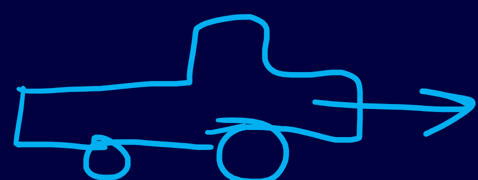
# Clicker Question 4:




- $W_{tot} = ½mv_f^2 - ½mv_i^2$
- $v_f = 0$ and $W_{tot} = -Fd$
- $-Fd = -½mv_i^2$

$$d = \frac{\frac{1}{2}mv_i^2}{F}$$

Since the faster arrow is twice as fast, the distance is $2^2 = 4$ times as long to stop it.

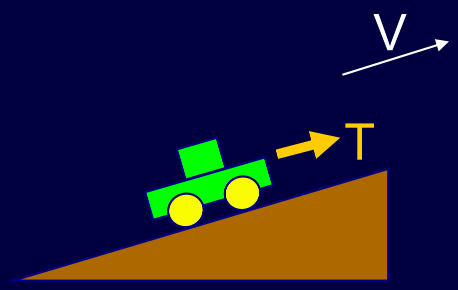


# Clicker Question 5:

You are towing a car up a hill with constant velocity.

The total work done on the car by all forces is:

A. positive
B. negative
C. zero



$W_{tot} = F_{net}\, d$

# Clicker Question 5:

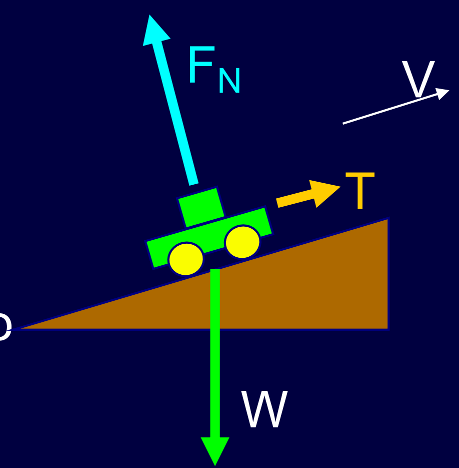
$a = 0$

You are towing a car up a hill with constant velocity. The total work done on the car by all forces is:

$F_{net} = 0$

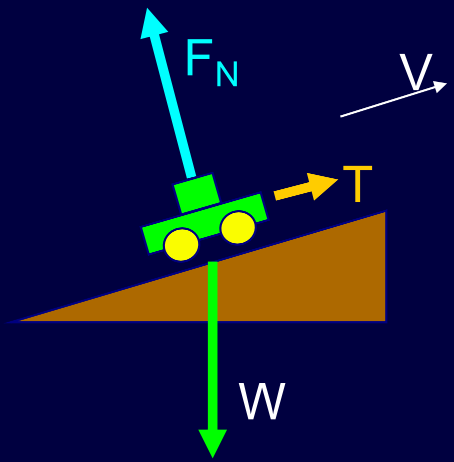
A. positive
B. negative
C. zero ⟵ correct

Since acceleration is zero, the x component of gravity and the force of tension must cancel each other out so the total work done by the car is zero.



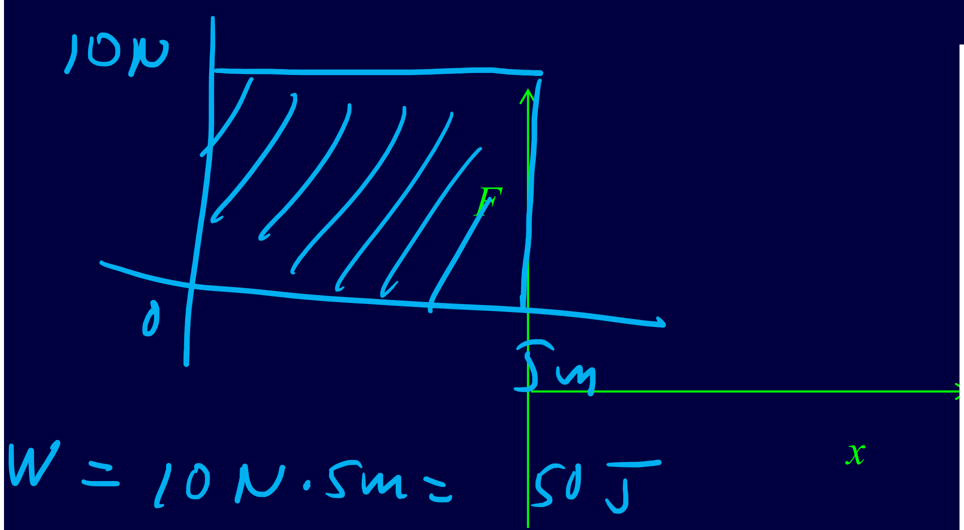
# Clicker Question 5:

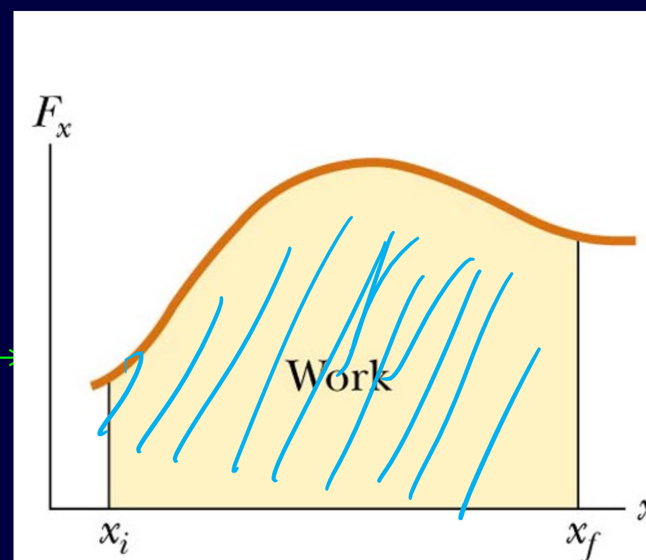
- (C) $W_{tot} = 0$
- $W_{tot} = ½mv_f^2 - ½mv_i^2$
- $v_f = v_i$
- So $½mv_f^2 - ½mv_i^2 = 0$
- $W_{tot} = ½mv_f^2 - ½mv_i^2 = 0$
- $W_T - W_g = 0$
- $W_T$ -+$W_g$



# Work by a Constant Force

- The work done by a force acting on an object that undergoes a displacement is equal to the area under the graph of F versus x



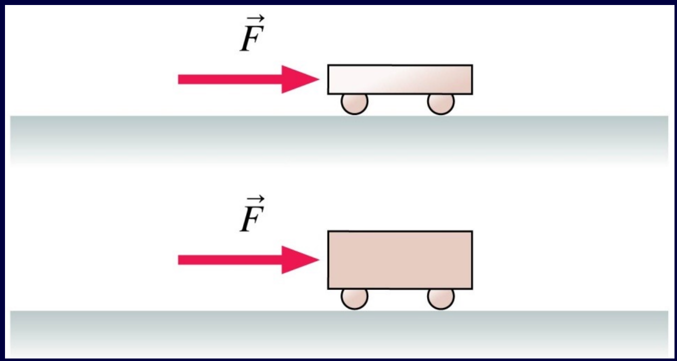
# Clicker Question 6:

A light plastic cart and a heavy steel cart are both pushed with the same force for a distance of $1.0$ m, starting from rest. After the force is removed, the kinetic energy of the light plastic cart is _________ that of the heavy steel cart.

$\frac{1}{2} m v^2$

$W = Fd \Rightarrow$ same

KE → same




$v_L$

$v_H$

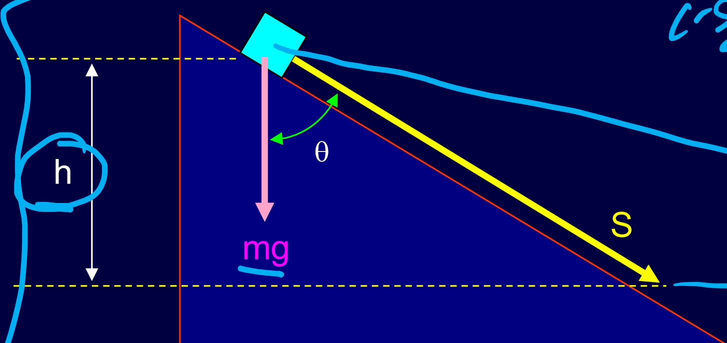
A. greater than
B. equal to
C. less than
D. Can't say. It depends on how big the force is.

$v_L > v_H$

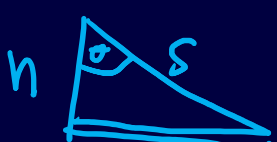
# Clicker Question 7:

- What is the work done by gravity as the block slides down the incline?

(a) W = mgh
(b) W = mgh*sin(θ)
(c) W = mgh*cos(θ)
(d) W = -mgh
(e) W = -mgh*sin(θ)

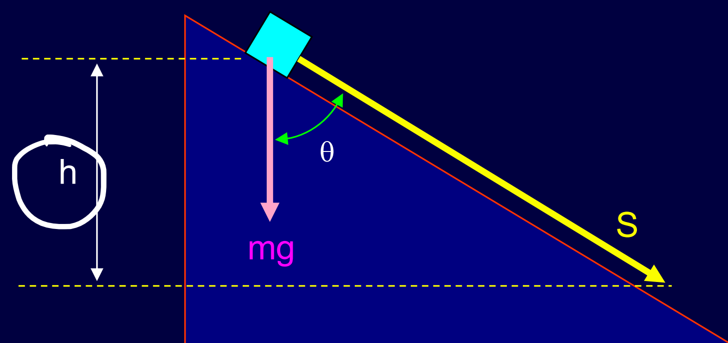



$$W = \vec{F}\cdot\vec{d} = |F||d|\cos\theta = mgS\cos\theta = mg\frac{h}{\cos\theta}\cos\theta = mgh$$




$$\cos\theta = \frac{A}{H} = \frac{h}{S} \qquad S = \frac{h}{\cos\theta}$$

# Clicker Question 7:




Gravity does zero work for motion in the x-direction

## Clicker Question 7:

You need to raise a heavy block by pulling it with a massless rope. You can either pull the block straight up height h, or pull it up a long, frictionless plane inclined at a 15 angle until its height has increased by h. Assume you will move the block at constant speed either way.

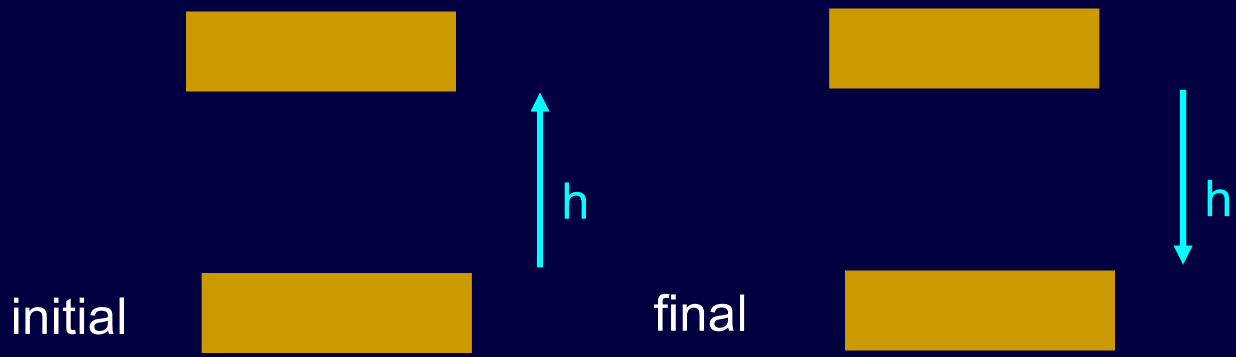
Student comments:

Correct: “The force exerted at a 15 degree angle is about 1/4 the force exerted by pulling up, however, the distance covered on the 15 degree ramp is about 4 times the distance covered by pulling straight up.

Correct:“in a) you exert more force over a shorter displacement and in b) you exert less force (due to the angle), over a longer displacement. since work is the product of these two quantities, it is the same in both cases”

## Clicker Question 8:

What total work does gravity do as my book is moved up and then back to it’s starting point?




(a) W = mgh
(b) W = 0
(c) W = -mgh