Lecture 9: Work and Energy

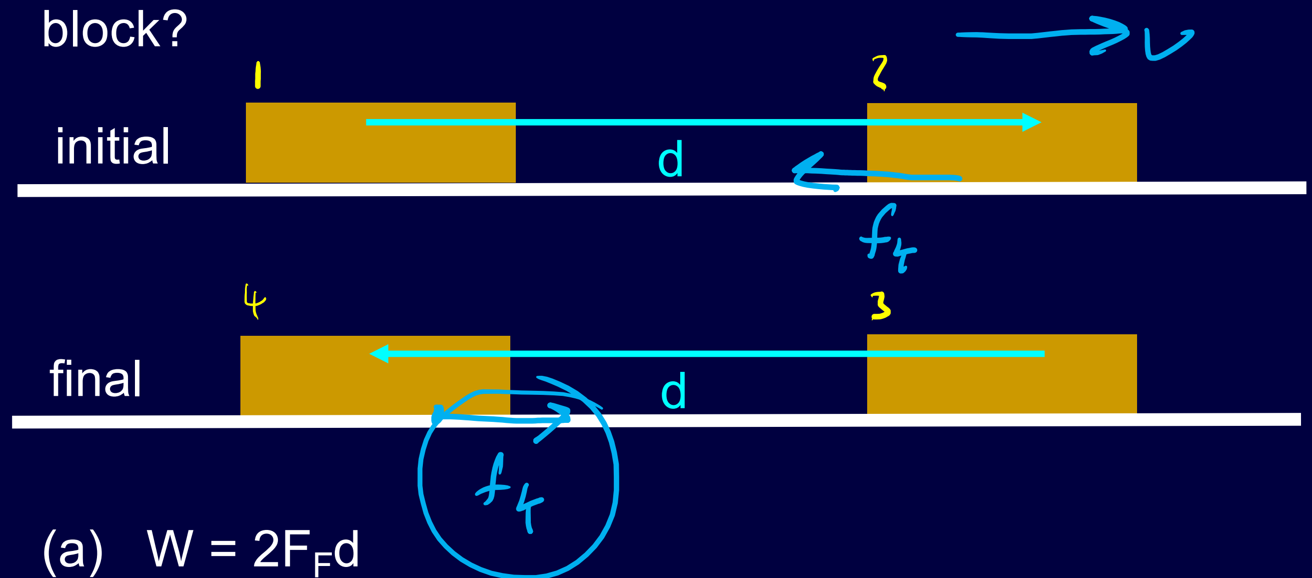
Today's Agenda

- Work and Energy
  - Definition of work
  - Examples
- Definition of Mechanical Energy
- Conservation of Mechanical Energy
  - Conservative forces

Start Recording

Clicker Question 0:

- A frictional force F_F resists the motion of the box below as it is moved to the right a distance d and then back to it's initial position. What total work does friction do on the block?



(a) $W = 2F_Fd$
(b) $W = 0$
(c) $W = -2F_Fd$

Potential energy

- Potential energy: Stored energy
- Energy depends on the position or configuration of an object.
- Potential energy due to gravity
- Potential energy of a spring.
- $\Delta U = -W_{conservative}$

$\Delta U = -W_{cons}$

$= -(-W)$

$\Delta U = W$

$W_c = -\Delta U$

Conservative forces

- For conservative forces the work done does not depend on path taken, only the starting and finishing points matter
  - Ex. gravity

- For conservative force work on a closed path is zero
  - When I move my book up and then down to the initial position work done by gravity was zero
- This semester
  - Conservative forces
    - Gravity
    - Springs
  - Non-conservative forces
    - Anything else!
- Conservative forces give object a potential energy!!!!

Conservation of Mechanical Energy

- If only conservative forces are doing work (ie W_{NC} is zero), the total mechanical energy of a system is conserved.

$W_{TOT} = \Delta K$

$W_{NC} + W_C = \Delta K$

$W_{NC} - \Delta U = \Delta K$

$W_{NC} = \Delta K + \Delta U$

$W_{NC} = E_f - E_i$

$$W_{NC} = E_f - E_i = 0 = 0$$

$$\boxed{E_{initial} = E_{final}}$$

E = *K* + *U* is ***constant!!!***

- Both *K* and *U* can change, but *E* = *K* + *U* remains constant.

Conservation of Mechanical Energy

- When only conservative forces are doing work on an object one can conserve mechanical energy

gravity

$E = K + U$

$$\boxed{E_{mech\,\mathrm{f}} = E_{mech\,\mathrm{i}}}$$

$$K_{\mathrm{f}} + U_{\mathrm{gf}} = K_{\mathrm{i}} + U_{\mathrm{gi}}$$

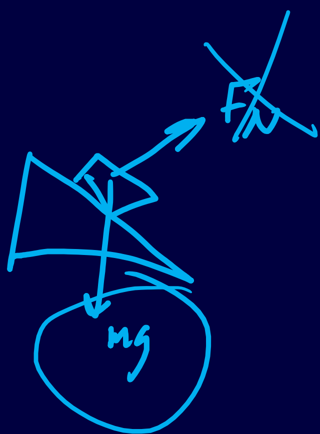
$$\tfrac{1}{2}mv_{\mathrm{f}}^2 + mgy_{\mathrm{f}} = \tfrac{1}{2}mv_{\mathrm{i}}^2 + mgy_{\mathrm{i}}$$

Mechanical energy is not always conserved!!!!!

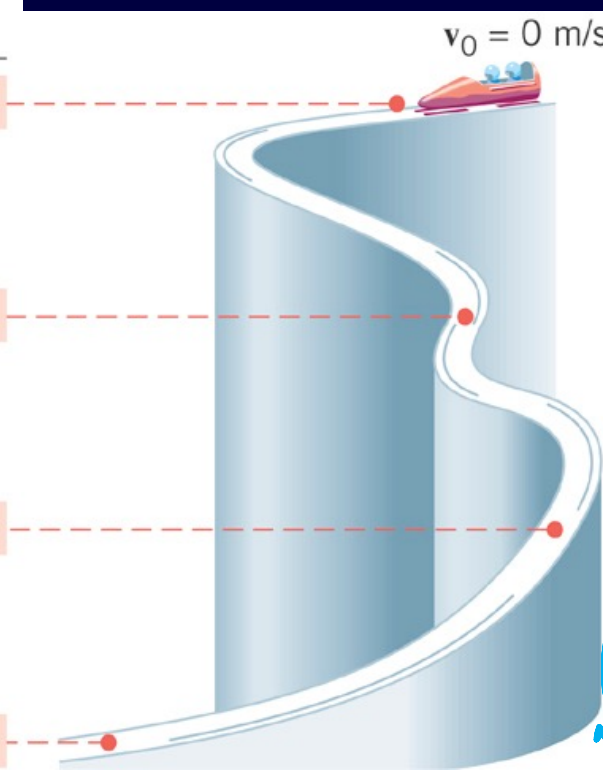
But Energy is.

Friction ≈ 0

Conservation of Mechanical Energy

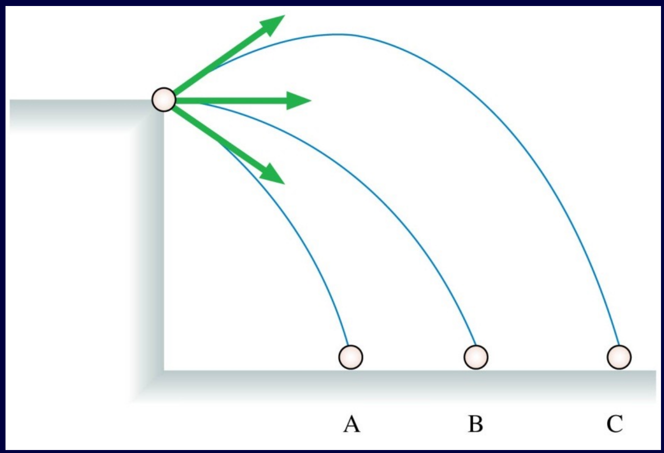


K	U	K + U
0 J	600 000 J	600 000 J
200 000 J	400 000 J	600 000 J
400 000 J	200 000 J	600 000 J
600 000 J	0 J	600 000 J



Clicker Question 0:

Three balls are thrown from a cliff with the same speed but at different angles. Which ball has the greatest speed just before it hits the ground?



A. Ball A.

B. Ball B.

C. Ball C.

D. All balls have the same speed.

E. Balls A and C

Clicker Question 0:

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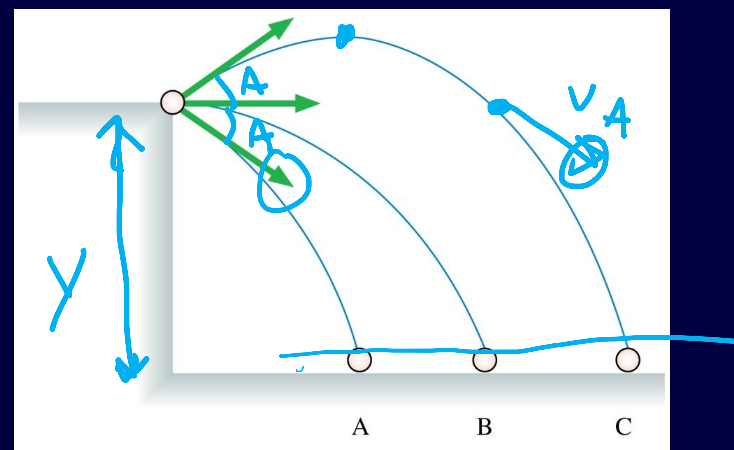


only gravity is doing work

$$\boxed{E_{\text{mech f}} = E_{\text{mech i}}}$$

$$K_{\text{f}} + U_{\text{gf}} = K_{\text{i}} + U_{\text{gi}}$$

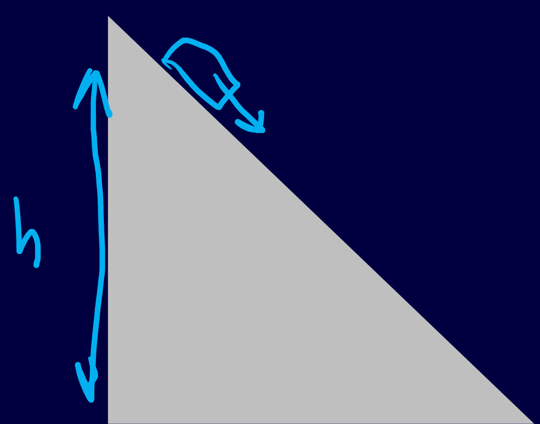
$$\tfrac{1}{2}\cancel{m}v_{\text{f}}^2 + \cancel{m}g(0) = \tfrac{1}{2}\cancel{m}v_{\text{i}}^2 + \cancel{m}gy$$ = same mech. energy

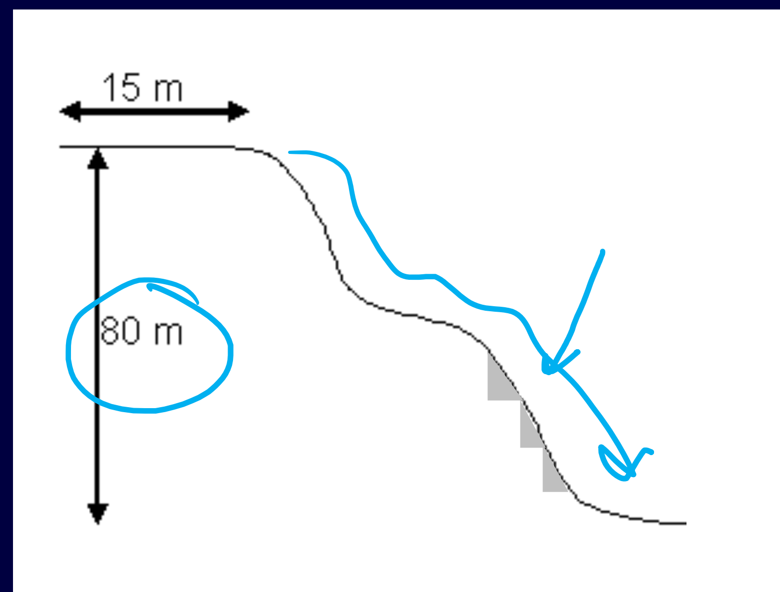
$$v_{\text{f}} = \sqrt{v_{\text{i}}^2 + 2gy}$$

$$v_{\text{f}}^2 = v_{\text{i}}^2 + 2gy$$

Student: why would we want to know if a force is conservative or non conservative?

Gravitational Potential Energy on a Frictionless Surface





Can approximate any surface as a bunch of small inclines!

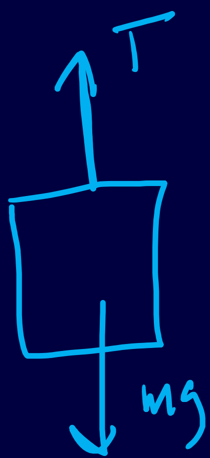
This conservation law works for any frictionless surface!

$$\Sigma \vec{F} = m\vec{a} = 0$$

$$W_{TOT} = \Delta K$$

$$W_{TOT} = 0$$

Clicker Question 1:



An elevator supported by a single cable descends a shaft at a constant speed. The only forces acting on the elevator are the tension in the cable and the gravitational force. Which one of the following statements is true?

$$W_{TOT} = \overset{0}{F_{net}}\, d = 0$$

a) The work done by the tension force is zero joules.

b) The net work done by the two forces is zero joules.

c) The work done by the gravitational force is zero joules.

d) The magnitude of the work done by the gravitational force is larger than that done by the tension force.

e) The magnitude of the work done by the tension force is larger than that done by the gravitational force.

Clicker Question 2:

$\downarrow mg$

A quarter is dropped from rest from the fifth floor of a very tall building. The speed of the quarter is *v* just before striking the ground. From what floor would the quarter have to be dropped from rest for the speed just before striking the ground to be approximately 2*v*? Ignore all air resistance effects to determine your answer.

$h_i = y$

y

$h_f = 0$

a) 14
b) 25
c) 20
d) 7
e) 10

$$E_{\text{mechf}} = E_{\text{mechi}}$$

$$K_{\text{f}} + U_{\text{gf}} = K_{\text{i}} + U_{\text{gi}}$$

$$\tfrac{1}{2}\cancel{m}v_{\text{f}}^2 + \boxed{\cancel{m}g(0)} = \tfrac{1}{2}\cancel{m(0)}^2 + \cancel{m}gy$$

$$\tfrac{1}{2}v_{\text{f}}^2 = gy$$

$$\boxed{v_{\text{f}} = \sqrt{2gy}}$$

$\sqrt{4} = 2$

Clicker Question 2:

A quarter is dropped from rest from the fifth floor of a very tall building. The speed of the quarter is *v* just before striking the ground. From what floor would the quarter have to be dropped from rest for the speed just before striking the ground to be approximately 2*v*?

Student Comments:

Correct: “since speed is squared in the kinetic energy equation, increasing the height by 4 will result in a two-fold increase in speed”

Incorrect : “By doubling the height, you are doubling the mechanical energy and therefore doubling the ending height.”

Kinetic Energy: Motion

- Apply constant force along x-axis to a point particle *m.*

  $W = F_x \Delta s$

  $= m\, a_x \Delta s$

  $= ½\, m\, (v_f^2 - v_i^2)$

$$\text{recall}: a_x \Delta s = \frac{1}{2}(v_{xf}^2 - v_{xi}^2)$$

- Work changes $½\, m\, v^2$

- This is the Kinetic Energy $K = ½\, m\, v^2$

$W = \Delta K$ For Point Particles

Work by a Non-Constant Force

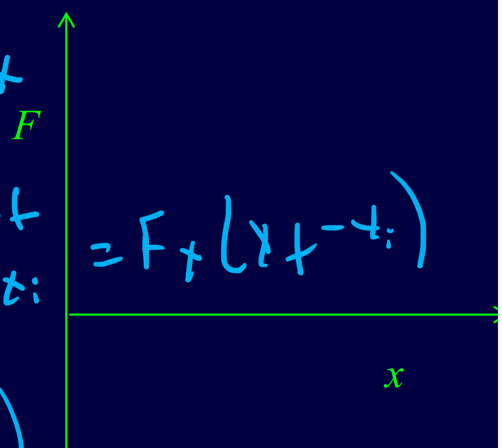
- The work done by a force acting on an object that undergoes a displacement is equal to the area under the graph of F versus x

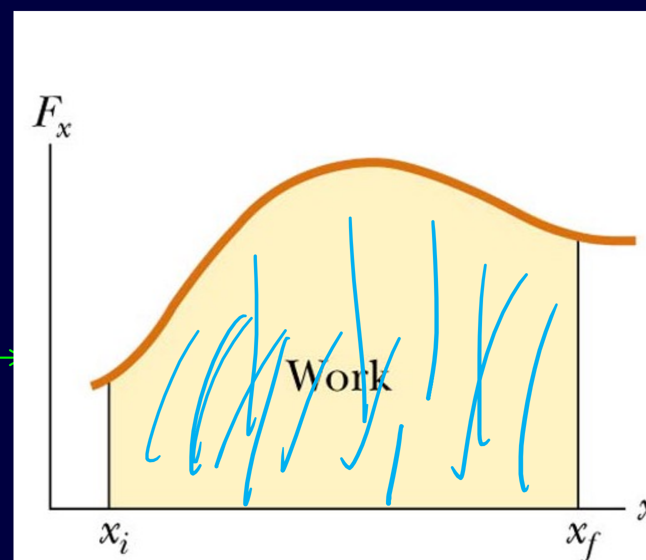
$$W = \int_{x_i}^{x_f} F_x \, dx$$

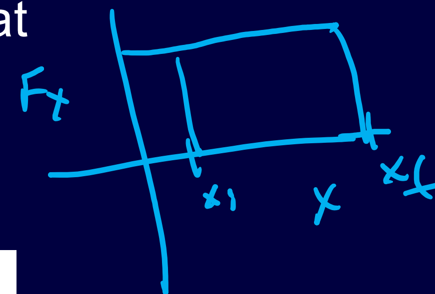
$$= F_x \int_{x_i}^{x_f} dx$$

$$= F_x \, x \Big]_{x_i}^{x_f} = F_x (x_f - x_i)$$

$$= F_x (\Delta x)$$







Work done by Gravity

$$W = \int \vec{F} \cdot d\vec{r} = \int \left(-mg\hat{y}\right) \cdot \left(dx\hat{x} + dy\hat{y} + dz\hat{z}\right)$$

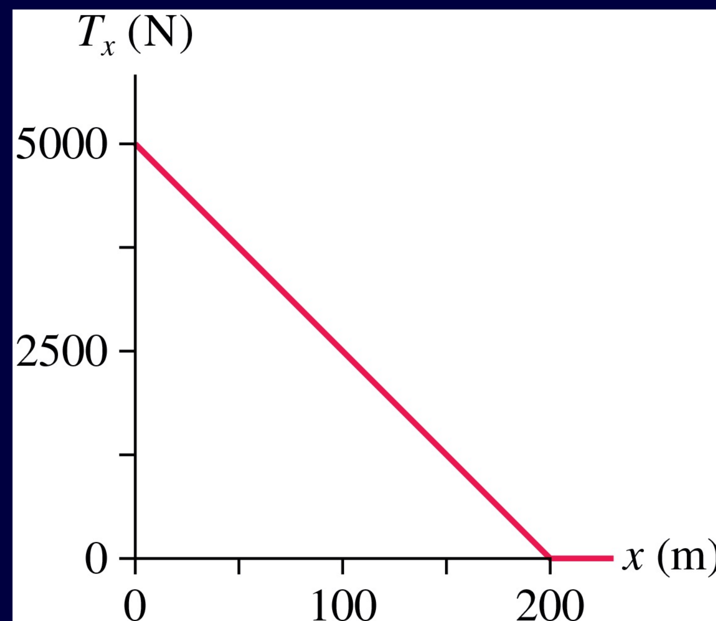
$$W = \int_{h_i}^{h_f} \left(-mg\right) dy = -mg\left(h_f - h_i\right)$$

$$\boxed{W = -mg\Delta h}$$

Independent of *x* and *z* directions

Clicker Question 3:

A 1500 kg car is towed, starting from rest. The figure below shows the tension force in the tow rope as the car travels from x = 0 m to x = 200 m. What is the car's speed after being pulled 200 m?



a) 15 m/s
b) 26 m/s
c) 20 m/s
d) 9 m/s
e) 11 m/s

Clicker Question 3:

A 1500 kg car is towed, starting from rest. The figure below shows the tension force in the tow rope as the car travels from x = 0 m to x = 200 m. What is the car's speed after being pulled 200 m?

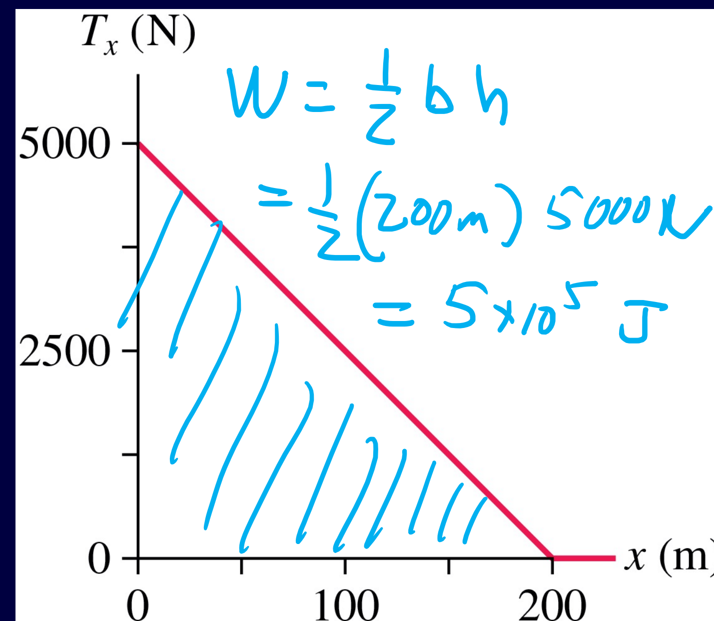
$$W_{T \, on \, T} = \Delta K$$

$$\int F_x \, dx = \frac{1}{2} m v_f^2 - \cancel{\frac{1}{2} m v_i^2}$$

$$\int F_x \, dx = \frac{1}{2} m v_f^2$$

$$\frac{2 \int F_x \, dx}{m} = v_f^2$$

$$v_f = \sqrt{\frac{2\left(\int F_x \, dx\right)}{m}} = \sqrt{\frac{2\left(5 \times 10^5 \, J\right)}{1500 \, kg}} = 26 \, m/s$$

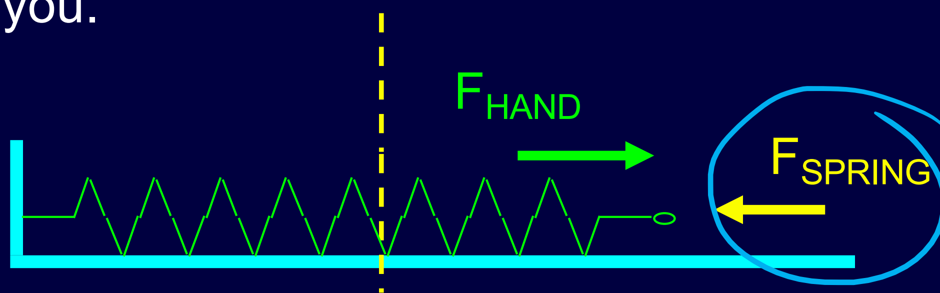


Force due to a spring

Happy place

- When to compress a spring the spring pushes out on you.

- When you stretch a spring the spring will pull back on you.



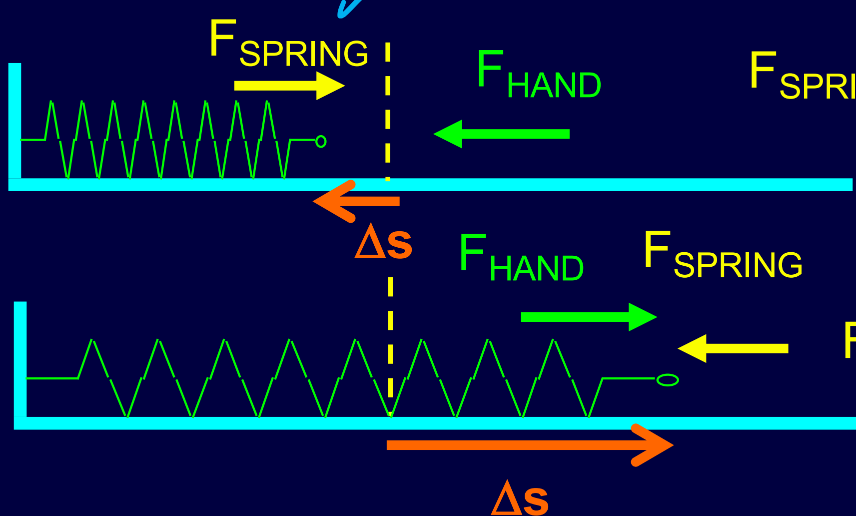
- Spring force always opposes force you apply

Student: The work of spring is in the opposite direction of the force acted upon it, which is similar to friction. So why is spring a conservative force but friction not a conservative force?

Force due to a spring

Hooke's Law

- $F_{SPRING} = -k(\Delta s)$
- Δs is distance spring is compressed or stretched from its relaxed length (happy place)
- k is the spring constant (how strong the spring is)



$F_{SPRING} = -k(-\Delta s) = +k\Delta s$

$F_{SPRING} = -k(+\Delta s) = -k\Delta s$

Spring force

ideal spring model

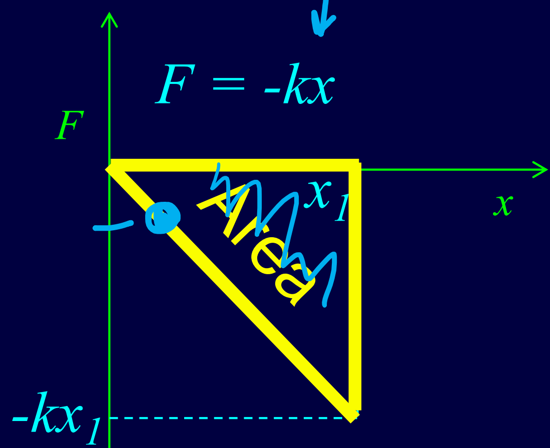
- Restoring force: Always works to restore the original position.

- The k constant depends on many factors (material, how many rings, etc) Only good for a particular spring
- Measure of stiffness
  - Large k real strong (shock absorber)
  - Small k not strong (slinky)

$U = \frac{1}{2} k x^2$

Work by a Non-constant Force

- The work done by a variable force acting on an object that undergoes a displacement is equal to the area under the graph of F versus x



$= \frac{1}{2} h b$

$W = ½ (-kx_1)(x_1)$

$W = -½ k(x_1)^2$

$W_C = -\Delta U$

$\boxed{\Delta U = ½ kx^2}$

$\Delta U = ½ k(\Delta s)^2$

Many objects can behave like a spring

- Restoring force
- Example
  - Bow and Arrow
  - Pendulum
  - Basketball
  - Superball
  - Tendons
  - Deforming an object
  - Many forces over small distances
  - Molecules

Conservation of Mechanical Energy

2A Good when only **gravity or a spring-like force are doing work on an object**

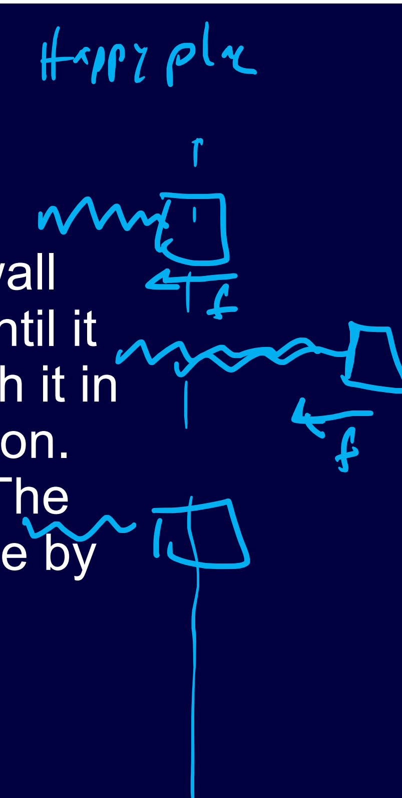
$$E_{mech\ f} = E_{mech\ i}$$

$$K_f + U_f = K_i + U_i$$

$$\tfrac{1}{2}k\Delta s_f^2 + \tfrac{1}{2}mv_f^2 + mgh_f = \tfrac{1}{2}k\Delta s_i^2 + \tfrac{1}{2}mv_i^2 + mgh_i$$

Clicker Question 4:

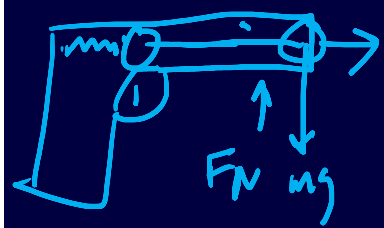
You grasp the end of a spring that is attached to the wall and is initially in its resting position. You pull it out until it is extended 0.1 m from its resting position, then push it in until it is compressed by 0.1 m from its resting position. Finally, you return the spring to its resting position. The spring constant is k = 20 N/m. The total work W done by the spring on your hand is



(a) $W < 0$
(b) $W = 0$
(c) $W > 0$

For conservative force work on a closed path is zero!

Clicker Question 5:



A spring-loaded gun shoots a plastic ball with a launch speed of 2.0 m/s. If the spring is then compressed twice the distance it was on the first shot. The ball's new launch speed will be

A. 1.0 m/s.

B. 2.0 m/s.

C. 2.8 m/s.

D. 4.0 m/s.

E. 16.0 m/s.

$$E_{\text{mech f}} = E_{\text{mech i}}$$

$$K_{\text{f}} + U_{\text{f}} = K_{\text{i}} + U_{\text{i}}$$

$$\tfrac{1}{2} m v_{\text{f}}^2 + \tfrac{1}{2} k (\Delta s)_{\text{f}}^2 = \underline{\tfrac{1}{2} m v_{\text{i}}^2} + \underline{\tfrac{1}{2} k (\Delta s)_{\text{i}}^2}$$

$$\cancel{\tfrac{1}{2}} m v_{\text{f}}^2 = \cancel{\tfrac{1}{2}} k (\Delta s)_{\text{i}}^2$$

$$\underline{v_{\text{f}}} = \underline{\Delta s}_i \sqrt{k / m}$$

Conservation of energy:
Double $\Delta x \Rightarrow$ increase v by factor of 2

Clicker Question 6:

→ gravity
→ Spring

- A spring is compressed by a ball an amount x = 0.5 m. The system is released and the ball is shot out down a frictionless table. At the end of the table the ball encounters a hump that brings it up to a height of 0.6 m. If the mass of the ball is 10 kg and the k for the spring is 600 N/m what is the final speed of the ball?

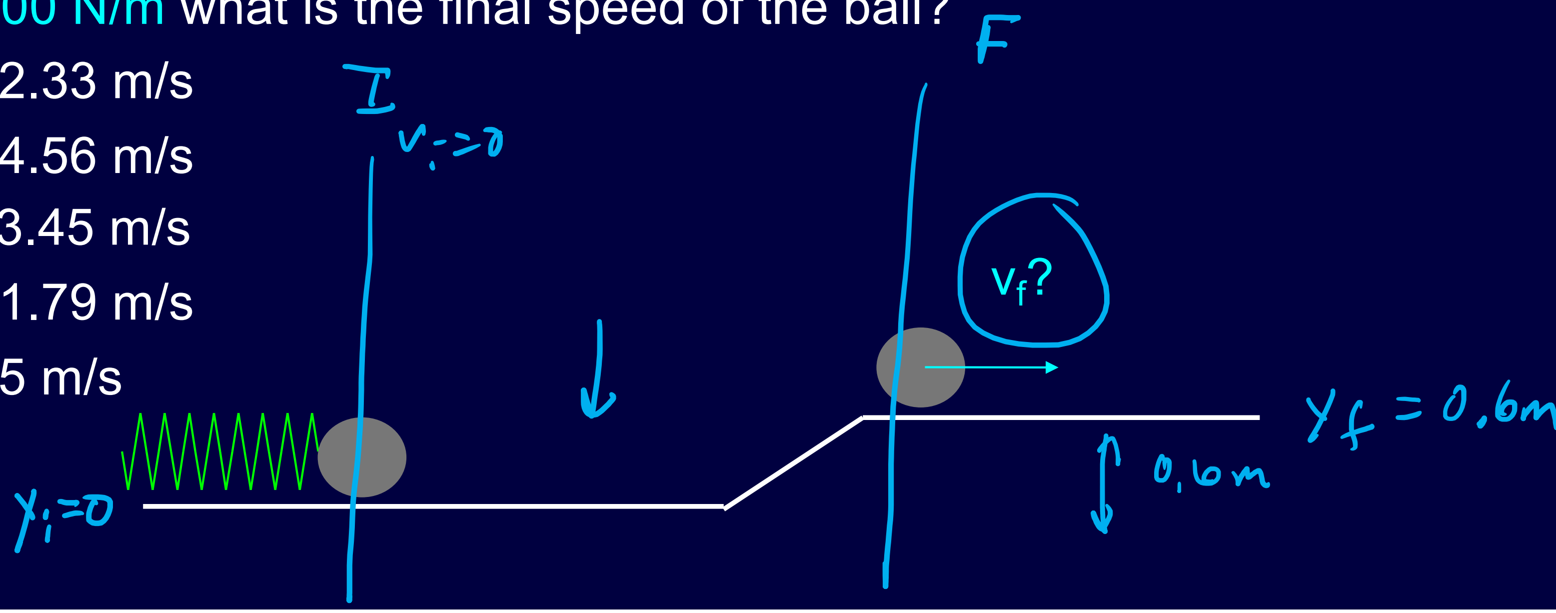
a) 2.33 m/s

b) 4.56 m/s

c) 3.45 m/s

d) 1.79 m/s

e) 5 m/s



$$E_f = E_i$$

$$K_f + U_f = K_i + U_i$$

$$\tfrac{1}{2}kx_f^2 + \tfrac{1}{2}mv_f^2 + mgy_f = \tfrac{1}{2}kx_i^2 + \tfrac{1}{2}mv_i^2 + mgy_i$$

$$\cancel{\tfrac{1}{2}k(0)^2} + \tfrac{1}{2}mv_f^2 + mg(.6) = \tfrac{1}{2}k(.5)^2 + \cancel{\tfrac{1}{2}m(0)^2} + \cancel{mg(0)}$$

$$\tfrac{1}{2}mv_f^2 + mg(.6) = \tfrac{1}{2}k(.5)^2$$

$$\tfrac{1}{2}mv_f^2 = \tfrac{1}{2}k(.5)^2 - mg(.6)$$

$$mv_f^2 = k(.5)^2 - 2mg(.6)$$

$$v_f^2 = (k/m)(.5)^2 - 2g(.6)$$

$$\boxed{v_f = 1.79 \text{ m/s}} \qquad v_f^2 = 3.22\ (\text{m/s})^2$$

Clicker Question 7:

The Jensens decided to spend their family vacation white water rafting. During one segment of their trip down a horizontal section of the river, the raft (total mass = 544 kg) has an initial speed of 6.75 m/s. The raft then drops a vertical distance of 14.0 m, ending with a final speed of 15.2 m/s. How much work was done on the raft by non-conservative forces?

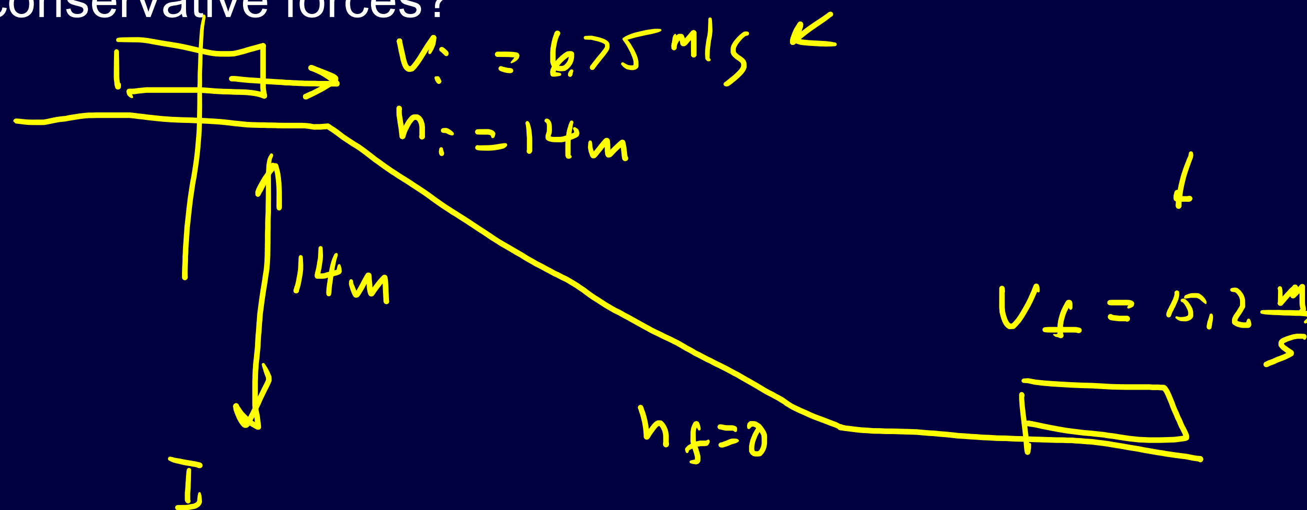
a) –12 100 J

b) –18 200 J

c) –24 200 J

d) –36 300 J

e) –48 400 J



Example Problems:

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$$W_{NC} = E_f - \underline{E_i} = -24{,}200\,J$$

$$= \left[\tfrac{1}{2}mv_f^2 + \cancel{mgh_f}\right] - \left[\tfrac{1}{2}mv_i^2 + mgh_i\right]$$

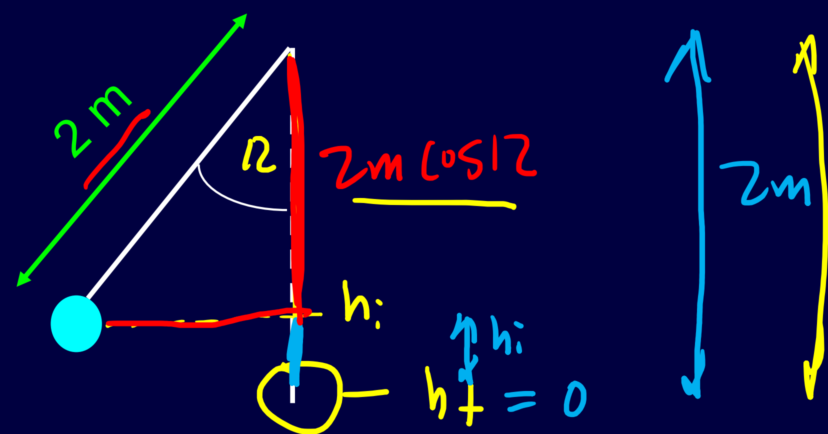
$$\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 - mgh_i$$

$$= \tfrac{1}{2}(544\,kg)(15.2\,m/s)^2 - \tfrac{1}{2}(544\,kg)(6.75\,m/s)^2 - 544\,g\,[14\,m]$$

$$= \underline{-24{,}200\,J}$$

Example Problems:

A pendulum is brought out an angle of 12°. What speed will it have when it returns to its initial position?



$$E_f = E_i$$

$$\frac{1}{2} m v_f^2 + mgh_f = \cancel{\frac{1}{2} m v_i^2} + mgh_i$$

$$\frac{1}{2} \cancel{m} v_f^2 + \cancel{m} g h_f = \cancel{m} g h_i$$

$$\frac{1}{2} v_f^2 + g h_f = g h_i$$

$$\frac{1}{2} v_f^2 = g h_i - g h_f$$

$$v_f = \sqrt{2g(h_i - h_f)}$$

$$= 0.928 \text{ m/s}$$

$$h_i = 2m - 2m \cos 12$$

$$= 0.044 m$$

A $\cos\theta$

B $\sin\theta$